

# BS2243 – Lecture 4

## Cournot duopoly and extensions

Spring 2012

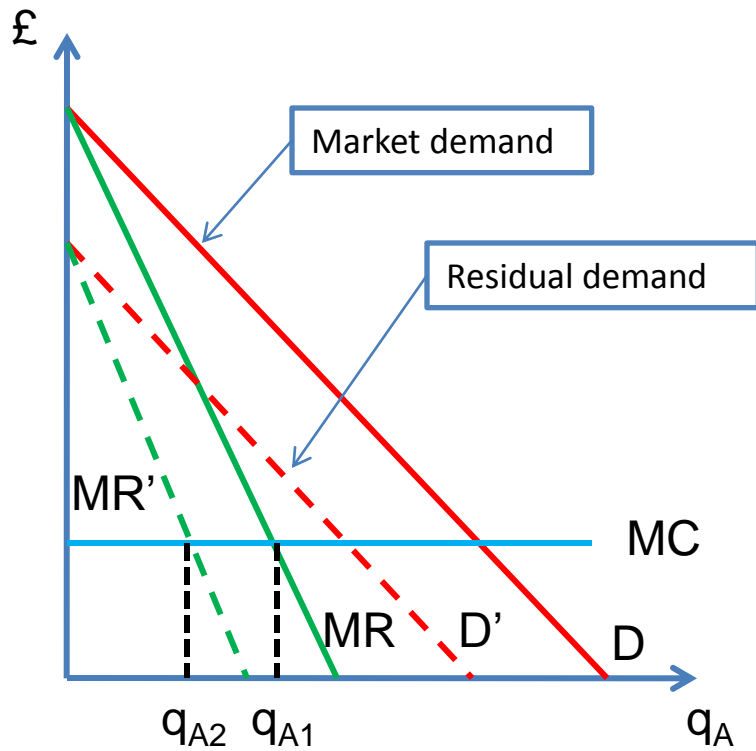
(Dr. Sumon Bhaumik)

# Cournot duopoly – market structure

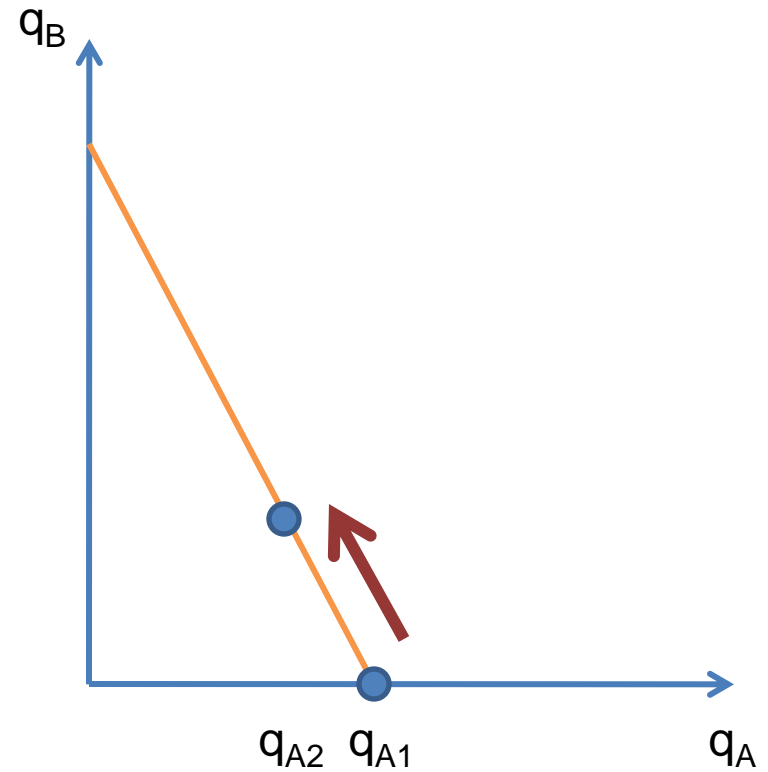
- Two firms (A and B)
  - Example: OPEC and non-OPEC oil producing countries
- Homogeneous product
- Competition in quantities
- Each firm assumes that the other firm will not react to its own choice of output

# Cournot duopoly – strategic behaviour

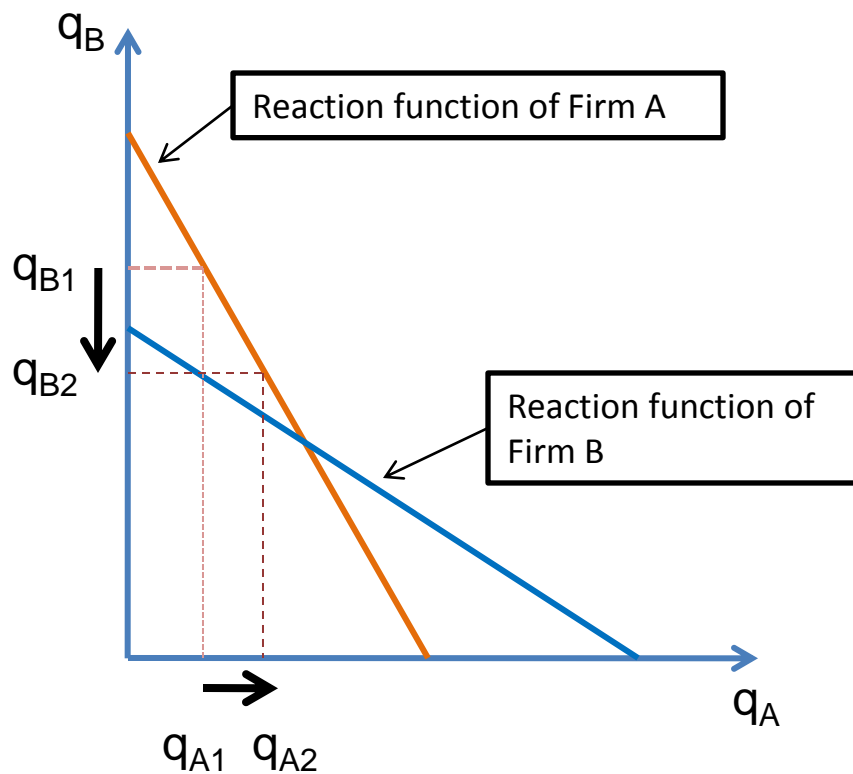
- Firm behaviour



- Reaction function



# Cournot duopoly – Nash equilibrium



- Each firm's output depends on the output choice of the other firm: Nash strategy
- At the quantity levels defined by the intersection of the two reaction functions, neither firm has any incentive to change output: equilibrium

# Algebra of Cournot duopoly - I

- Inverse demand curve

$$P = 1000 - 10Q$$

- Two identical firms

– Firm 1 produces  $q_1$  and Firm 2 produces  $q_2$

$$q_1 + q_2 = Q$$

- Cost structure

$$AC = MC = 50$$

# Algebra of Cournot duopoly - II

- Profit maximising condition for a firm

$$MC = MR$$

- Decision

– How much to produce?

- Rewriting inverse demand curve

$$P = 1000 - 10(q_1 + q_2)$$

$$P = 1000 - 10q_1 - 10q_2$$

- Marginal revenue curve

$$\text{Firm 1: } (1000 - 10q_2) - 20q_1$$

$$\text{Firm 2: } (1000 - 10q_1) - 20q_2$$

# Algebra of Cournot duopoly - III

- Profit maximisation

Firm 1:  $(1000 - 10q_2) - 20q_1 = 50$

$$20q_1 + 10q_2 = 950 \quad (\text{Verify: } q_1 = 47.5 - 0.5q_2)$$

Reaction function of Firm 1

Firm 2:  $(1000 - 10q_1) - 20q_2 = 50$

$$10q_1 + 20q_2 = 950 \quad (\text{Verify: } q_2 = (95/2) - 0.5q_1)$$

Reaction function of Firm 2

- Nash equilibrium

Solve the reaction functions simultaneously

$$20q_1 + 10q_2 = 950$$

$$10q_1 + 20q_2 = 950$$

# Algebra of Cournot duopoly - IV

- Quantities in equilibrium

Solving the reaction functions simultaneously

$$q_1 = \quad , q_2 =$$

- Price in equilibrium

$$P = 1000 - 10(q_1 + q_2) =$$

- Profits in equilibrium

$$\pi_1 = (P - AC) \times q_1 =$$

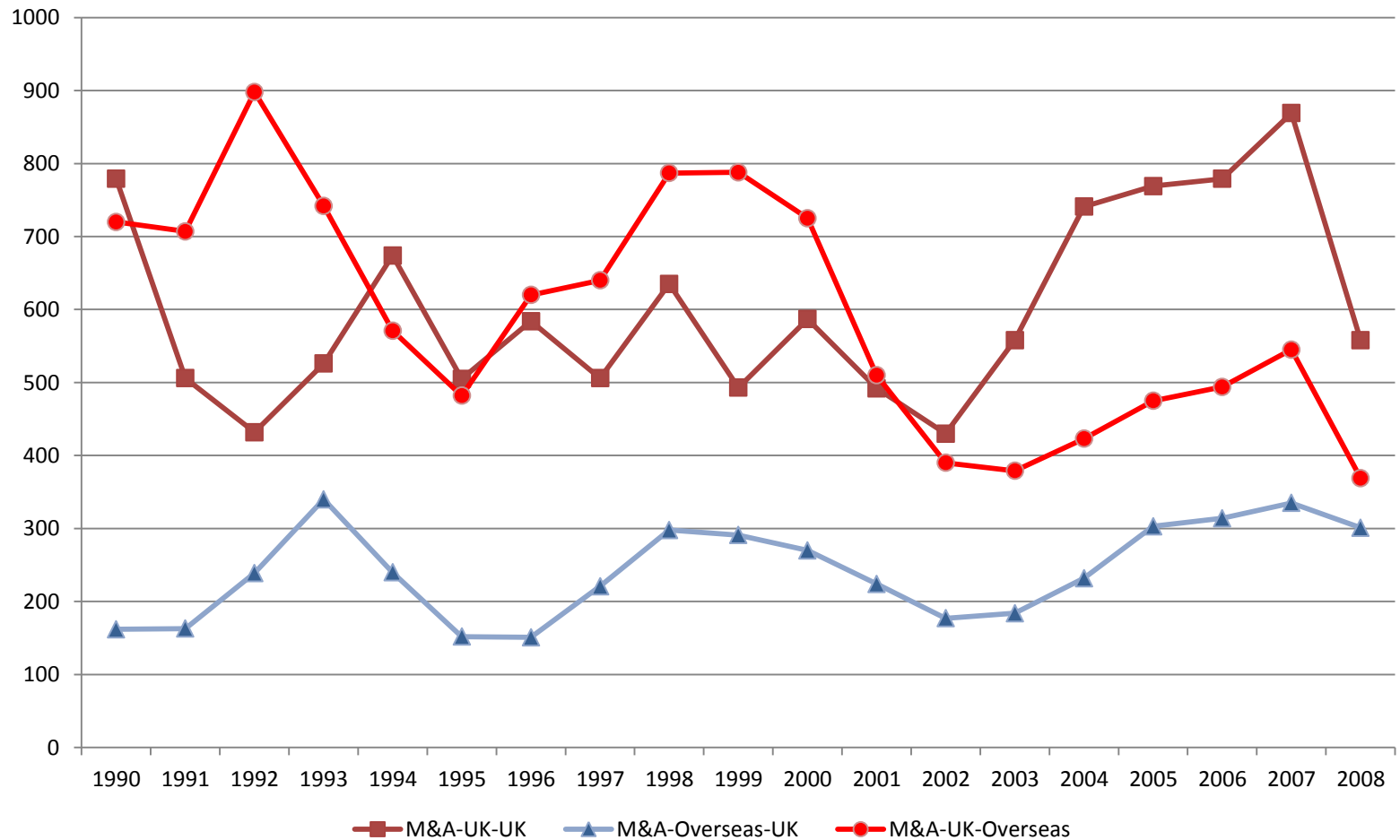
$$\pi_2 = (P - AC) \times q_2 =$$



# Strategy I – merger or collusion

- Market effectively has one multi-plant firm
  - Firm 1 has become Plant 1, and Firm 2 has become Plant 2
- Decisions
  - How much to produce?
  - How to distribute the output between the two plants?

# Strategy I – incidence of merger



Source: Office of National Statistics

# Strategy I – intuition

- The multi-plant firm will set output at the level where  $MC = MR$
- It will allocate a larger share of the output to the firm with the lower cost
- If the plants have identical cost structures, the optimum output will be equally divided between the two plants

# Algebra for Strategy I – I

- Inverse demand curve  
 $P = 1000 - 10Q$
- Two identical plants
  - Plant 1 produces  $q_1$  and Plant 2 produces  $q_2$   
 $q_1 = q_2 = Q/2$
- Cost structure  
 $AC = MC = 50$
- Profit maximising condition for a firm  
 $MC = MR$   
 $50 = 1000 - 20Q$

# Algebra for Strategy I – II

- Decisions

- How much to produce?

$$1000 - 20Q = 50$$

$$20Q = 950$$

$$Q = 47.5$$

- How to distribute output between the two plants?

$$q_1 = q_2 = Q/2 = 47.5/2 = 23.75$$

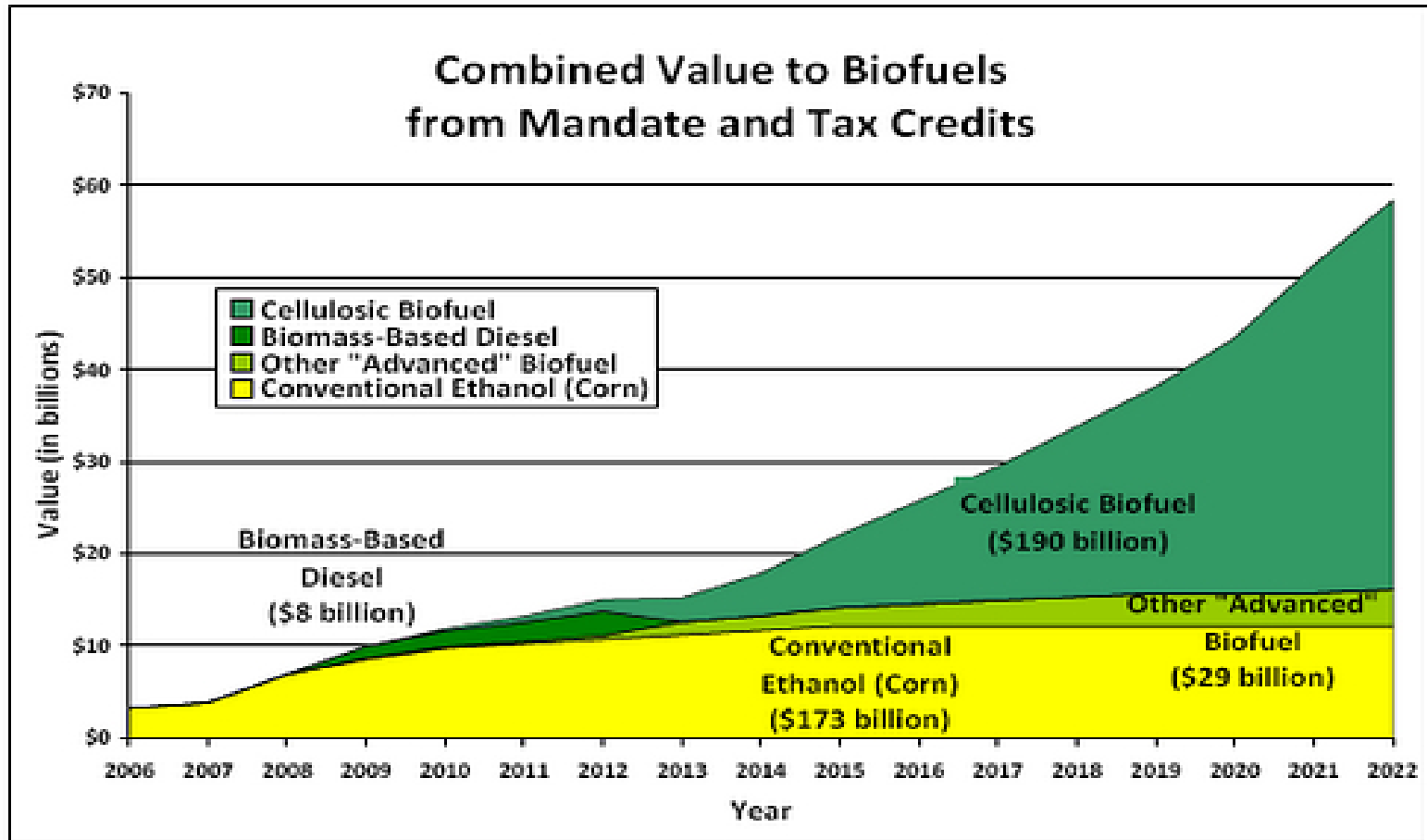
- Outcomes

$$P = 1000 - 10Q = 1000 - (10 \times 47.5) = 525$$

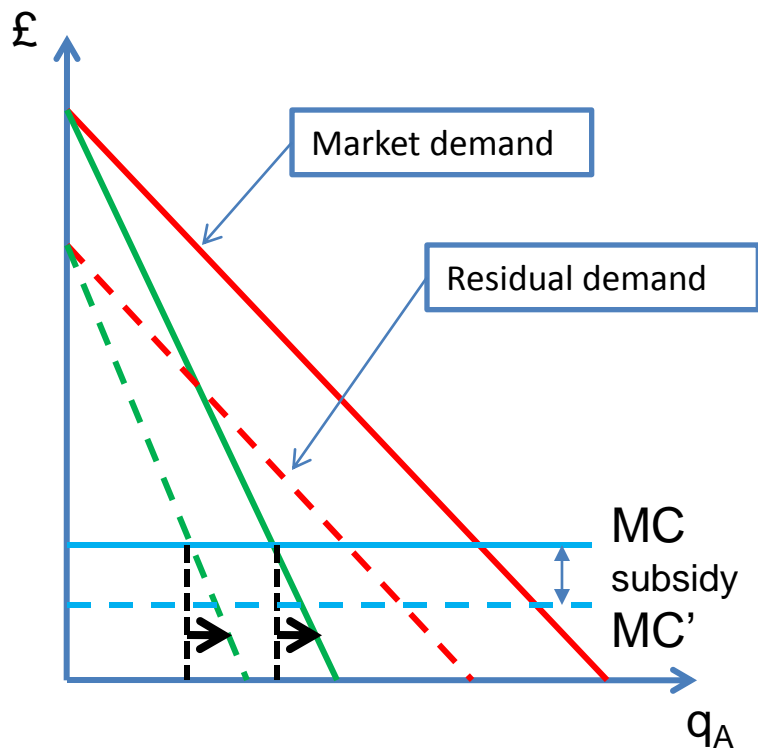
$$\pi = (P - AC) \times Q = (525 - 50) \times 47.5 =$$

In case of collusion, profit shared equally by the two firms

# Strategy II – lobby for subsidy



# Strategy II – impact of subsidy



- Subsidy reduces marginal cost of production
- The new marginal cost equals the marginal revenue at a higher output level
- The optimum output level of the firm is higher

# Algebra for Strategy II – I

- Inverse demand curve  
 $P = 1000 - 10Q$
- Two identical firms
  - Firm 1 produces  $q_1$  and Firm 2 produces  $q_2$   
 $q_1 + q_2 = Q$
- Firm 1 gets a subsidy of 10 per unit of output
- Cost structure  
Firm 1:  $AC = MC = 50 - 10 = 40$   
Firm 2:  $AC = MC = 50$



# Algebra of Strategy II - II

- Profit maximising condition for a firm

$$MC = MR$$

- Decision

– How much to produce?

- Rewriting inverse demand curve

$$P = 1000 - 10(q_1 + q_2)$$

$$P = 1000 - 10q_1 - 10q_2$$

- Marginal revenue curve

$$\text{Firm 1: } (1000 - 10q_2) - 20q_1$$

$$\text{Firm 2: } (1000 - 10q_1) - 20q_2$$

# Algebra of Strategy II - III

- Profit maximisation

Firm 1:  $(1000 - 10q_2) - 20q_1 = 40$   
 $20q_1 + 10q_2 = 960$  (Verify:  $q_1 = 48 - 0.5q_2$ )  
Reaction function of Firm 1

Firm 2:  $(1000 - 10q_1) - 20q_2 = 50$   
 $10q_1 + 20q_2 = 950$   
Reaction function of Firm 2

- Nash equilibrium

Solve the reaction functions simultaneously

$$20q_1 + 10q_2 = 960$$

$$10q_1 + 20q_2 = 950$$

# Algebra of Strategy II - IV

- Quantities in equilibrium

Solving the reaction functions simultaneously

$$q_1 = \quad , q_2 =$$

- Price in equilibrium

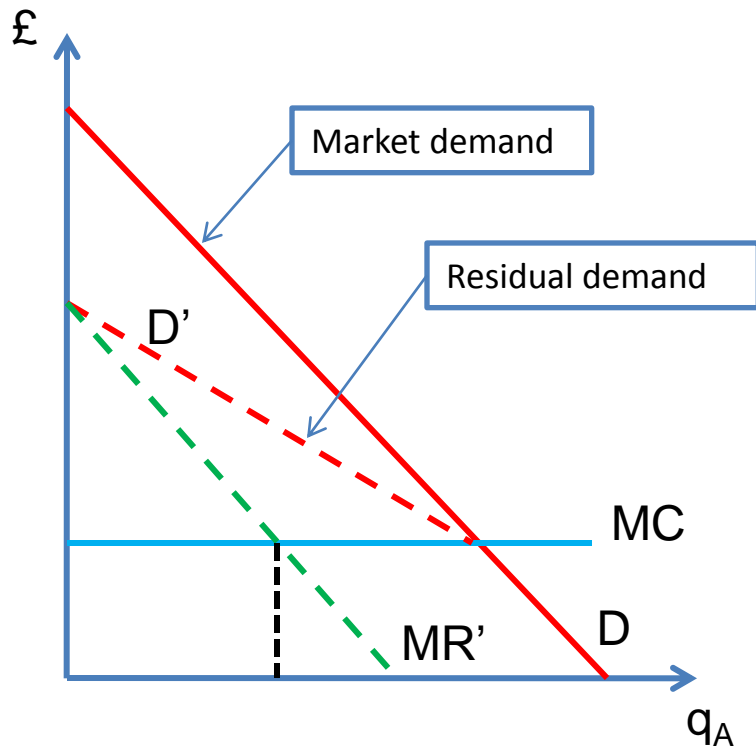
$$P = 1000 - 10(q_1 + q_2) =$$

- Profits in equilibrium

$$\pi_1 = (P - AC) \times q_1 =$$

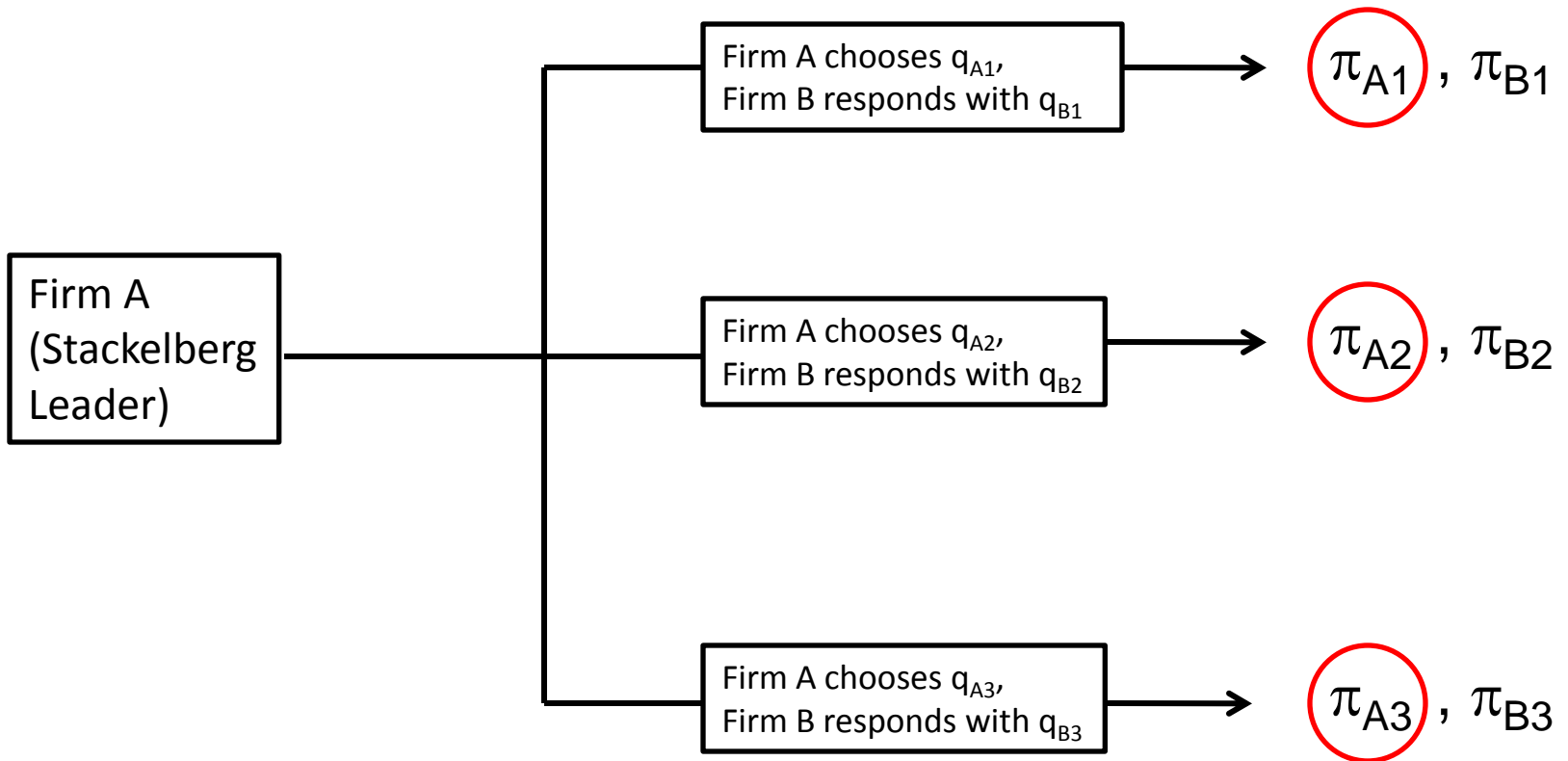
$$\pi_2 = (P - AC) \times q_2 =$$

# Strategy III – become a Stackelberg leader



- Firm A (the Stackelberg leader) takes the strategic behaviour of Firm B into consideration
- Note the difference in the residual demand curve (relative to the Cournot competition scenario)
- In equilibrium, Firm A (the leader) would be better off and Firm B (the follower) would be worse off

# Strategy III – become a Stackelberg leader



# From concept to algebra

## Cournot duopoly

- Each firm naively maximises profits by setting  $MC = MR$
- Profit maximisation gives us the reaction functions of the firms
- We then have two equations (reaction functions) with two unknowns ( $q_1$  and  $q_2$ )
- Plugging the quantities into the demand function gives us the price
- The price, costs and quantities together give us the profits

## Stackelberg duopoly

- The follower (Firm 2) naively maximises profits by setting  $MC = MR$
- Profit maximisation gives us the reaction function of the follower
- The leader (Firm 1) takes the follower's reaction function into consideration when it decides on its residual demand curve
- The leader's profit maximisation gives us  $q_1$
- Using this in the reaction function of the follower gives us  $q_2$
- We get price and profits as in Cournot

# Algebra of Stackelberg duopoly - I

- Inverse demand curve  
 $P = 1000 - 10Q$
- Two identical firms
  - Firm 1 produces  $q_1$  and Firm 2 produces  $q_2$   
 $q_1 + q_2 = Q$
  - Firm 1 is Stackelberg leader
- Cost structure  
 $AC = MC = 50$

# Algebra of Stackelberg duopoly - II

- Profit maximisation of Firm 2

$$(1000 - 10q_1) - 20q_2 = 50 \quad (\text{from the algebra of Cournot})$$

$$10q_1 + 20q_2 = 950 \quad (\text{reaction function of Firm 2})$$

$$q_2 = (950 - 10q_1)/20 = 47.5 - 0.5q_1$$

- Demand function of Firm 1

$$P = 1000 - 10q_2 - 10q_1$$

$$= 1000 - 10(47.5 - 0.5q_1) - 10q_1 = 525 - 5q_1$$

- Profit maximisation of Firm 1

$$525 - 10q_1 = 50$$



# Algebra of Stackelberg duopoly - III

- Quantities in equilibrium

First solve the profit maximisation problem of Firm 1

$$q_1 =$$

Then substitute  $q_1$  into the reaction function of Firm 2

$$q_2 =$$

- Price in equilibrium

$$P = 1000 - 10(q_1 + q_2) =$$

- Profits in equilibrium

$$\pi_1 = (P - AC) \times q_1 =$$

$$\pi_2 = (P - AC) \times q_2 =$$