BS2243 – Lecture 4
Cournot duopoly and extensions

Spring 2012

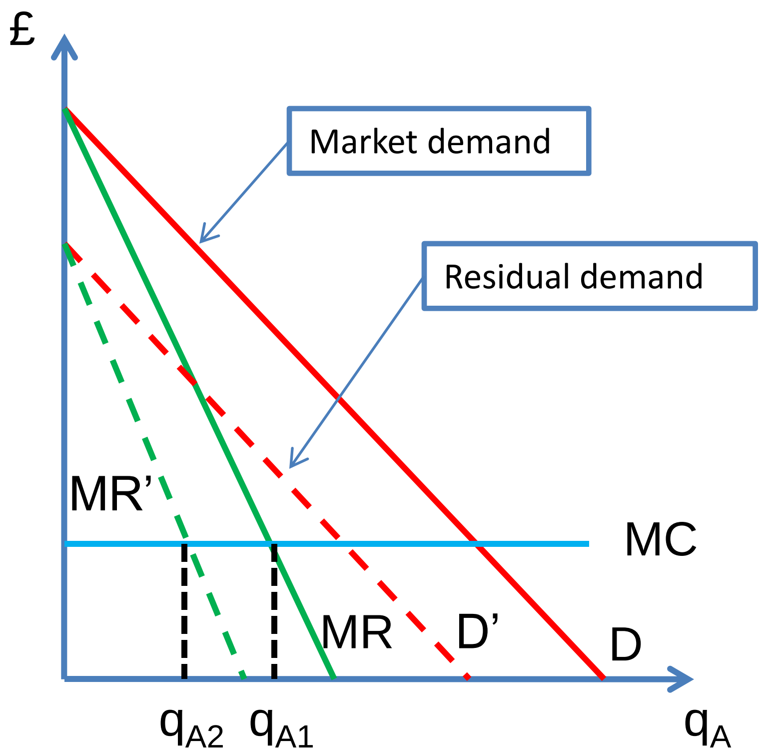
(Dr. Sumon Bhaumik)

Cournot duopoly – market structure

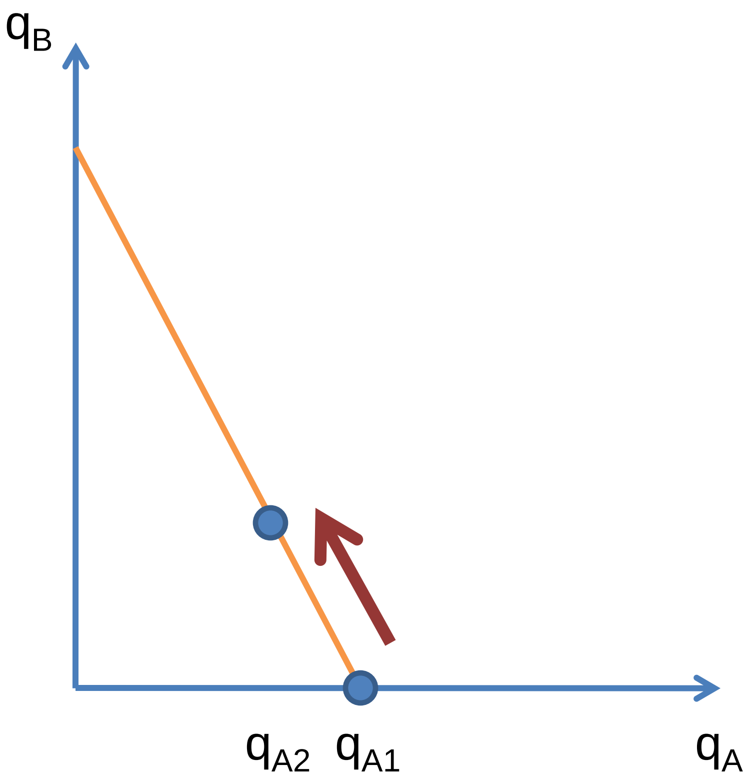
- Two firms (A and B)
  - Example: OPEC and non-OPEC oil producing countries
- Homogeneous product
- Competition in quantities
- Each firm assumes that the other firm will not react to its own choice of output

Cournot duopoly – strategic behaviour

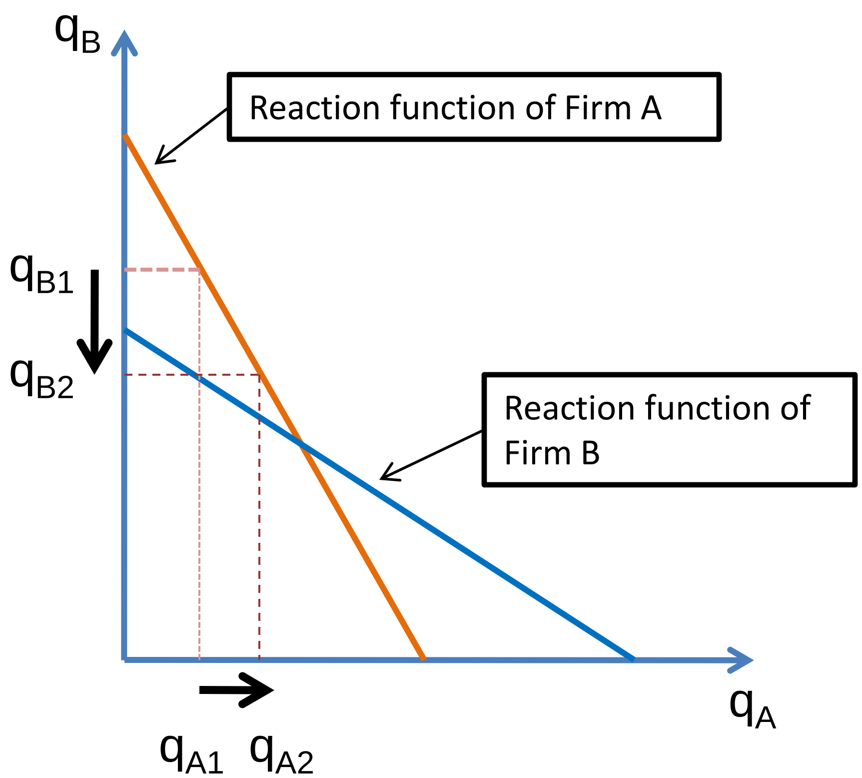
- Firm behaviour



- Reaction function



Cournot duopoly – Nash equilibrium



- Each firm's output depends on the output choice of the other firm: <u>Nash strategy</u>

- At the quantity levels defined by the intersection of the two reaction functions, neither firm has any incentive to change output: <u>equilibrium</u>

Algebra of Cournot duopoly - I

- Inverse demand curve

  $P = 1000 - 10Q$

- Two identical firms
  - Firm 1 produces q_1 and Firm 2 produces q_2

    $q_1 + q_2 = Q$

- Cost structure

  $AC = MC = 50$

Algebra of Cournot duopoly - II

- Profit maximising condition for a firm
  MC = MR

- Decision
  - How much to produce?

- Rewriting inverse demand curve
  $P = 1000 – 10(q_1 + q_2)$
  $P = 1000 – 10q_1 – 10q_2$

- Marginal revenue curve
  Firm 1: $(1000 – 10q_2) – 20q_1$
  Firm 2: $(1000 – 10q_1) – 20q_2$

Algebra of Cournot duopoly - III

- Profit maximisation

  Firm 1: $(1000 – 10q_2) – 20q_1 = 50$

  $20q_1 + 10q_2 = 950$ (Verify: $q_1 = 47.5 – 0.5q_2$)

  Reaction function of Firm 1

  Firm 2: $(1000 – 10q_1) – 20q_2 = 50$

  $10q_1 + 20q_2 = 950$ (Verify: $q_2 = (95/2) – 0.5q_1$)

  Reaction function of Firm 2

- Nash equilibrium

  Solve the reaction functions simultaneously

  $20q_1 + 10q_2 = 950$

  $10q_1 + 20q_2 = 950$

Algebra of Cournot duopoly - IV

- Quantities in equilibrium
  Solving the reaction functions simultaneously
  $q_1 =$, $q_2 =$

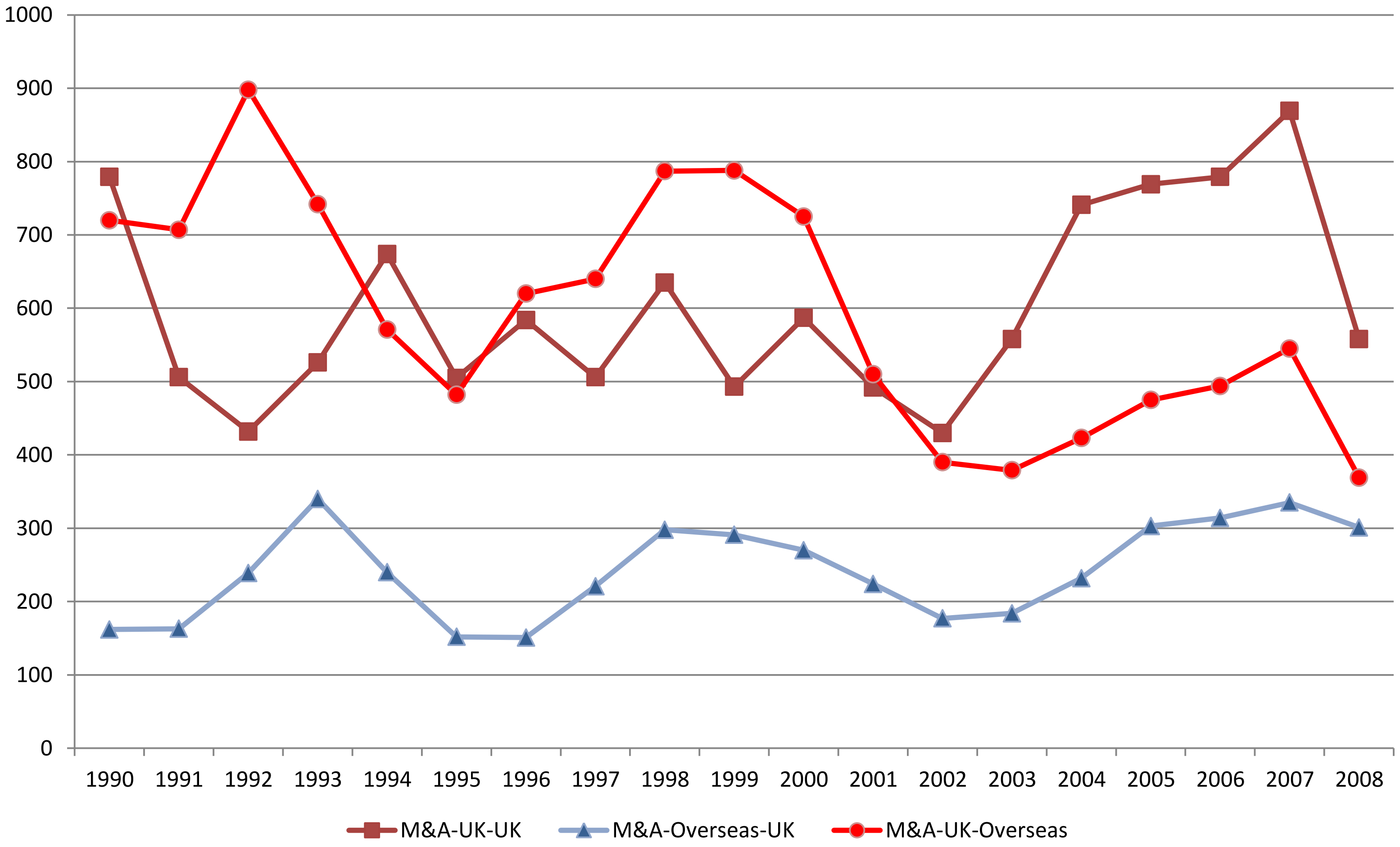
- Price in equilibrium
  $P = 1000 – 10(q_1 + q_2) =$

- Profits in equilibrium
  $\pi_1 = (P – AC) \times q_1 =$
  $\pi_2 = (P – AC) \times q_2 =$

Strategy I – merger or collusion

- Market effectively has one multi-plant firm
  - Firm 1 has become Plant 1, and Firm 2 has become Plant 2
- Decisions
  - How much to produce?
  - How to distribute the output between the two plants?

Strategy I – incidence of merger



Source: Office of National Statistics

Strategy I – intuition

- The multi-plant firm will set output at the level where MC = MR

- It will allocate a larger share of the output to the firm with the lower cost

- If the plants have identical cost structures, the optimum output will be equally divided between the two plants

Algebra for Strategy I – I

- Inverse demand curve
  $P = 1000 - 10Q$

- Two identical plants
  - Plant 1 produces q_1 and Plant 2 produces q_2
    $q_1 = q_2 = Q/2$

- Cost structure
  $AC = MC = 50$

- Profit maximising condition for a firm
  $MC = MR$
  $50 = 1000 - 20Q$

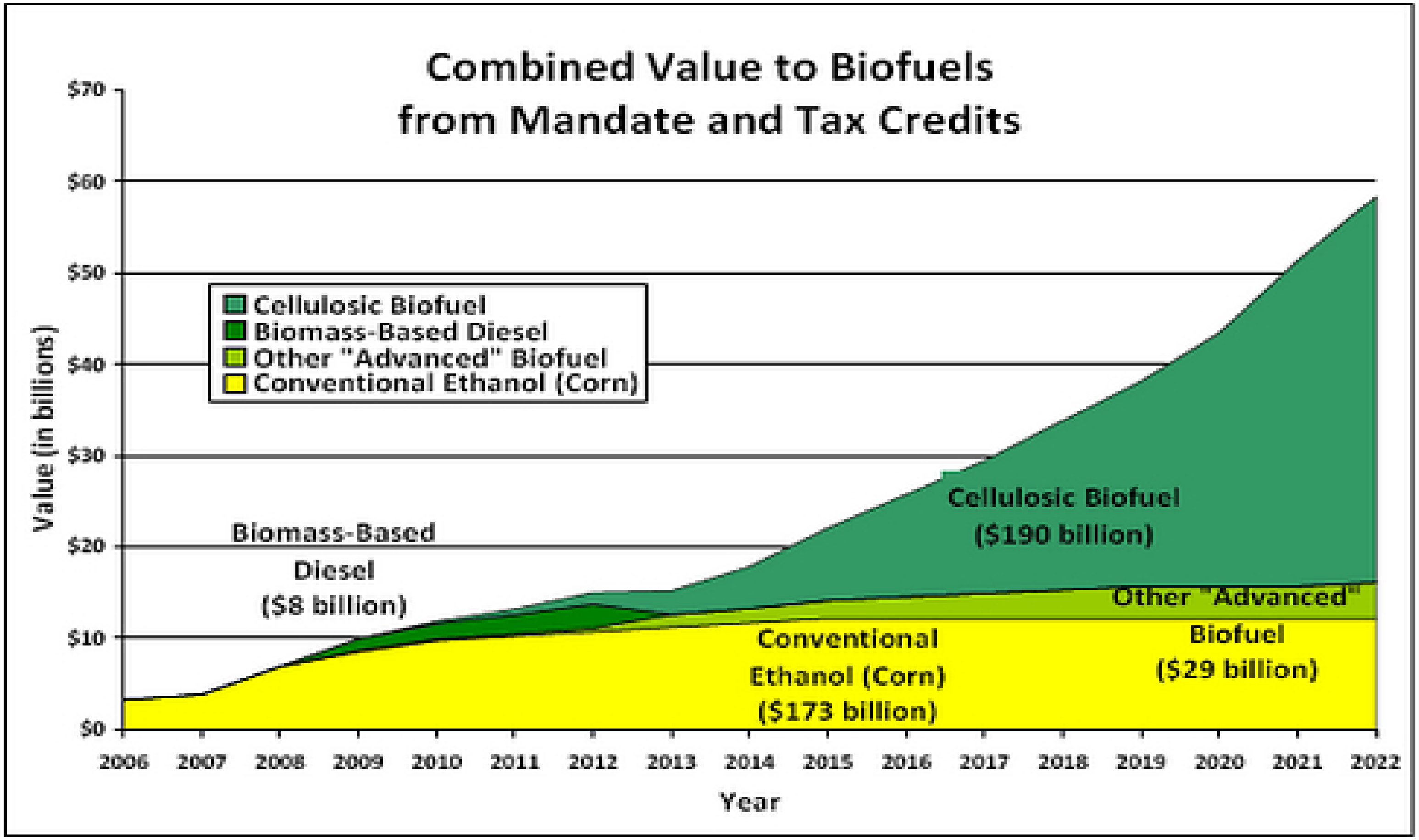
Algebra for Strategy I – II

- Decisions
  - How much to produce?
    $1000 - 20Q = 50$
    $20Q = 950$
    $Q = 47.5$

  - How to distribute output between the two plants?
    $q_1 = q_2 = Q/2 = 47.5/2 = 28.75$

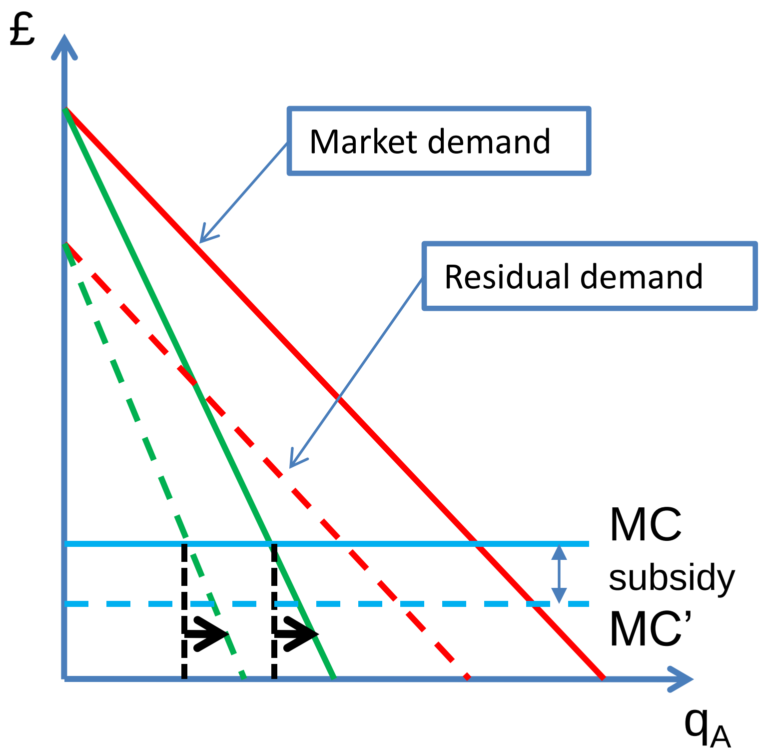
- Outcomes
  $P = 1000 - 10Q = 1000 - (10 \times 47.5) = 525$
  $\pi = (P - AC) \times Q = (525 - 50) \times 47.5 =$
  In case of collusion, profit shared equally by the two firms

Strategy II – lobby for subsidy



http://blogs.wsj.com/environmentalcapital/2009/05/07/biofuels-bill-federal-subsidies-will-top-400-billion-enviros-say/tab/article/

Strategy II – impact of subsidy



- Subsidy reduces marginal cost of production
- The new marginal cost equals the marginal revenue at a higher output level
- The optimum output level of the firm is higher

Algebra for Strategy II – I

- Inverse demand curve
  $P = 1000 - 10Q$

- Two identical firms
  - Firm 1 produces q_1 and Firm 2 produces q_2
    $q_1 + q_2 = Q$

- Firm 1 gets a subsidy of 10 per unit of output

- Cost structure
  Firm 1: $AC = MC = 50 - 10 = 40$
  Firm 2: $AC = MC = 50$

Algebra of Strategy II - II

- Profit maximising condition for a firm
  MC = MR

- Decision
  - How much to produce?

- Rewriting inverse demand curve
  $P = 1000 – 10(q_1 + q_2)$
  $P = 1000 – 10q_1 – 10q_2$

- Marginal revenue curve
  Firm 1: $(1000 – 10q_2) – 20q_1$
  Firm 2: $(1000 – 10q_1) – 20q_2$

Algebra of Strategy II - III

- Profit maximisation

  Firm 1: $(1000 – 10q_2) – 20q_1 = 40$

  $20q_1 + 10q_2 = 960$ (Verify: $q_1 = 48 – 0.5q_2$)

  Reaction function of Firm 1

  Firm 2: $(1000 – 10q_1) – 20q_2 = 50$

  $10q_1 + 20q_2 = 950$

  Reaction function of Firm 2

- Nash equilibrium

  Solve the reaction functions simultaneously

  $20q_1 + 10q_2 = 960$

  $10q_1 + 20q_2 = 950$

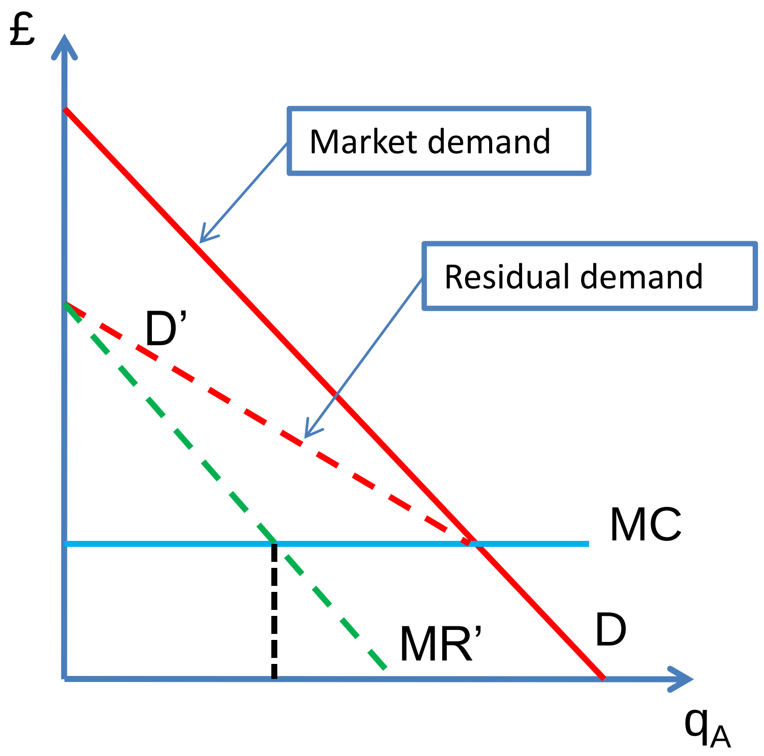
Algebra of Strategy II - IV

- Quantities in equilibrium
  Solving the reaction functions simultaneously
  $q_1 =$, $q_2 =$

- Price in equilibrium
  $P = 1000 – 10(q_1 + q_2) =$

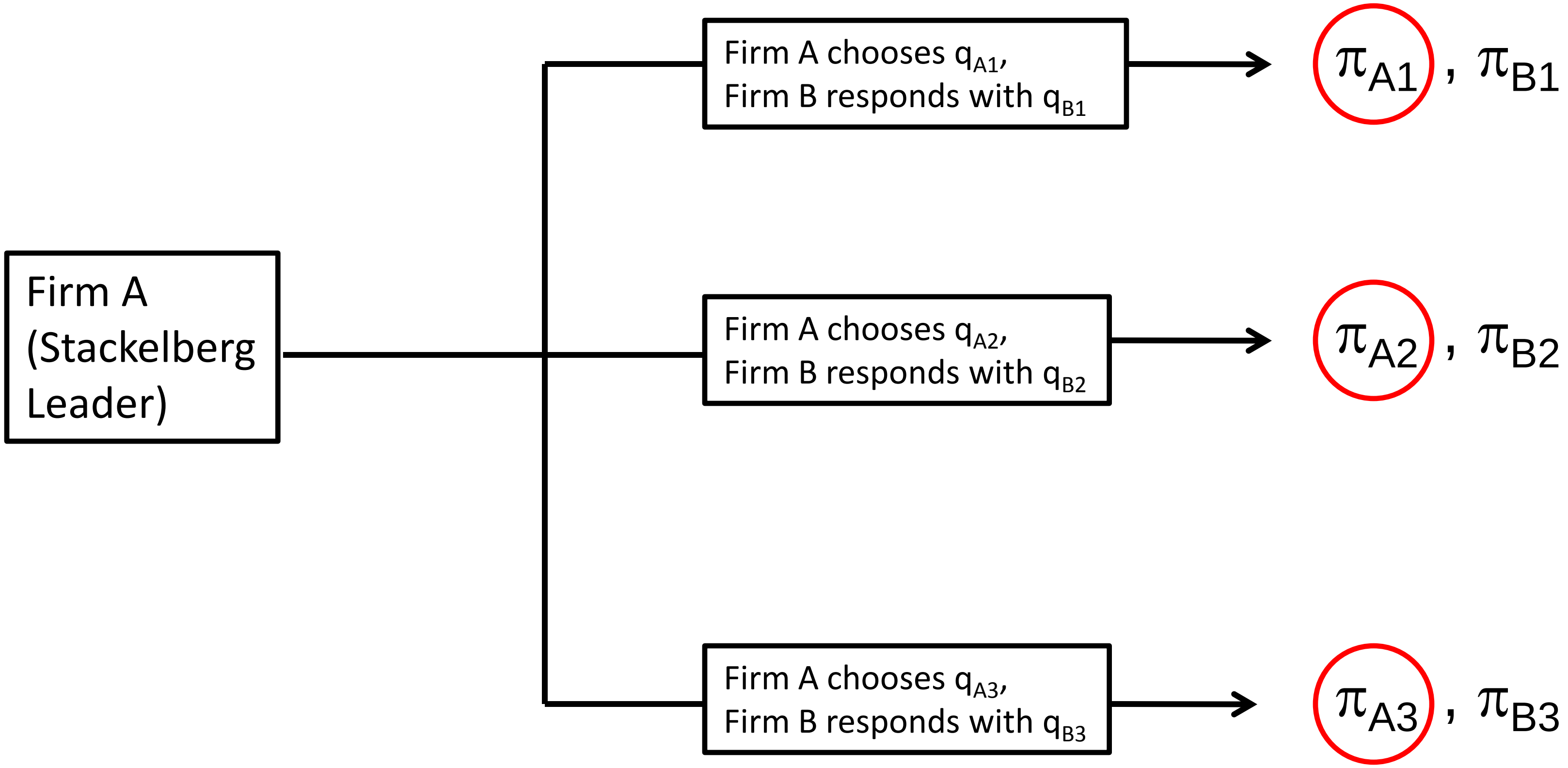
- Profits in equilibrium
  $\pi_1 = (P – AC) \times q_1 =$
  $\pi_2 = (P – AC) \times q_2 =$

Strategy III – become a Stackelberg leader



- Firm A (the Stackelberg leader) takes the strategic behaviour of Firm B into consideration
- Note the difference in the residual demand curve (relative to the Cournot competition scenario)
- In equilibrium, Firm A (the leader) would be better off and Firm B (the follower) would be worse off

Strategy III – become a Stackelberg leader



From concept to algebra

Cournot duopoly

- Each firm naively maximises profits by setting MC = MR
- Profit maximisation gives us the reaction functions of the firms
- We then have two equations (reaction functions) with two unknowns (q_1 and q_2)
- Plugging the quantities into the demand function gives us the price
- The price, costs and quantities together give us the profits

Stackelberg duopoly

- The follower (Firm 2) naively maximises profits by setting MC = MR
- Profit maximisation gives us the reaction function of the follower
- The leader (Firm 1) takes the follower's reaction function into consideration when it decides on its residual demand curve
- The leader's profit maximisation gives us q_1
- Using this in the reaction function of the follower gives us q_2
- We get price and profits as in Cournot

Algebra of Stackelberg duopoly - I

- Inverse demand curve
  $P = 1000 - 10Q$

- Two identical firms
  - Firm 1 produces q_1 and Firm 2 produces q_2
    $q_1 + q_2 = Q$
  - Firm 1 is Stackelberg leader

- Cost structure
  $AC = MC = 50$

Algebra of Stackelberg duopoly - II

- Profit maximisation of Firm 2

$(1000 – 10q_1) – 20q_2 = 50$ (from the algebra of Cournot)

$10q_1 + 20q_2 = 950$ (reaction function of Firm 2)

$q_2 = (950 – 10q_1)/20 = 47.5 – 0.5q_1$

- Demand function of Firm 1

$P = 1000 – 10q_2 – 10q_1$

$= 1000 – 10(47.5 - 0.5q_1) – 10q_1 = 525 – 5q_1$

- Profit maximisation of Firm 1

$525 – 10q_1 = 50$

Algebra of Stackelberg duopoly - III

- Quantities in equilibrium
  First solve the profit maximisation problem of Firm 1
  q_1 =
  Then substitute q_1 into the reaction function of Firm 2
  q_2 =

- Price in equilibrium
  $P = 1000 – 10(q_1 + q_2) =$

- Profits in equilibrium
  $\pi_1 = (P – AC) \times q_1 =$
  $\pi_2 = (P – AC) \times q_2 =$