

BS2243 – Lecture 5

Cournot duopoly and extensions

Spring 2010

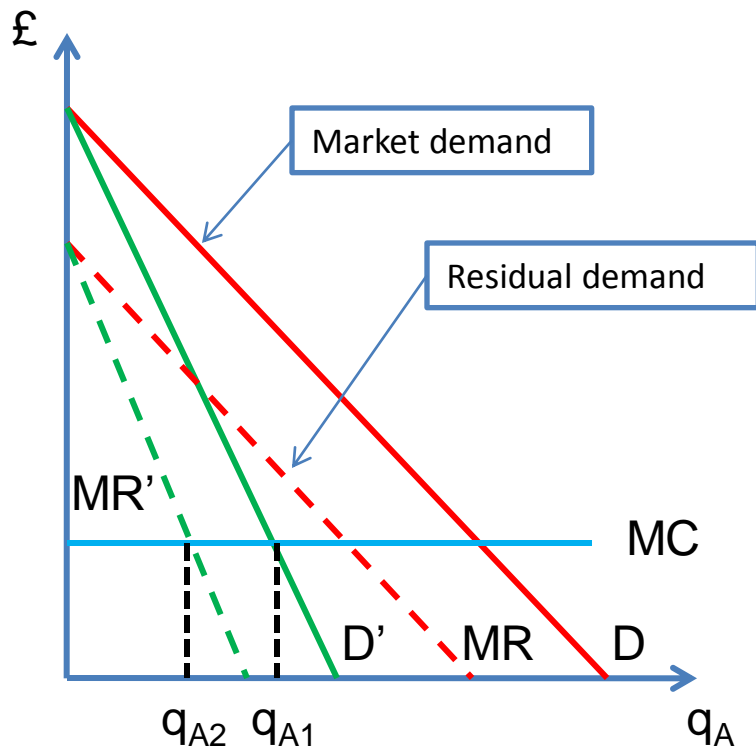
(Dr. Sumon Bhaumik)

Cournot duopoly – market structure

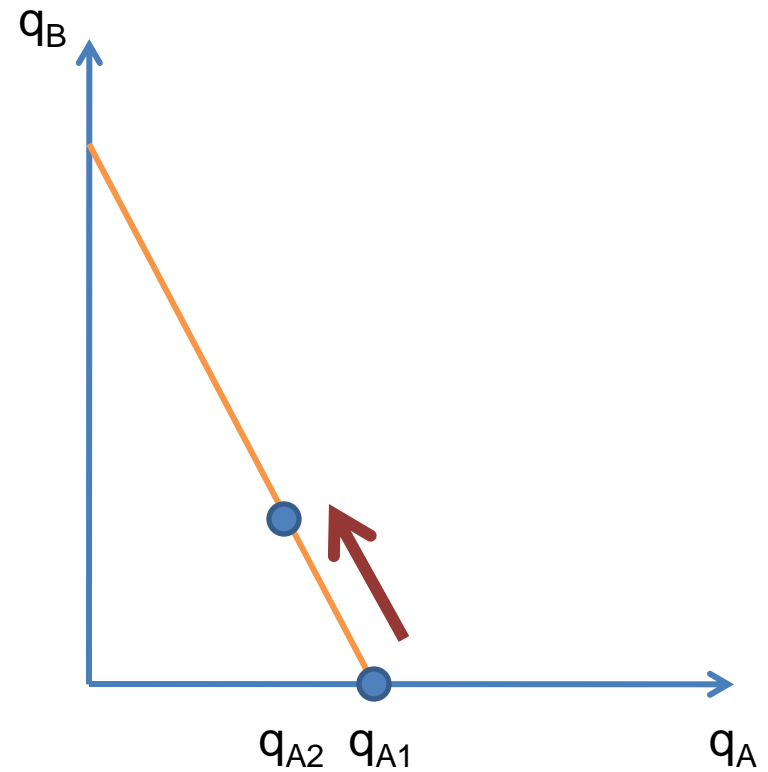
- Two firms (A and B)
 - Example: OPEC and non-OPEC oil producing countries
- Homogeneous product
- Competition in quantities
- Each firm assumes that the other firm will not react to its own choice of output

Cournot duopoly – strategic behaviour

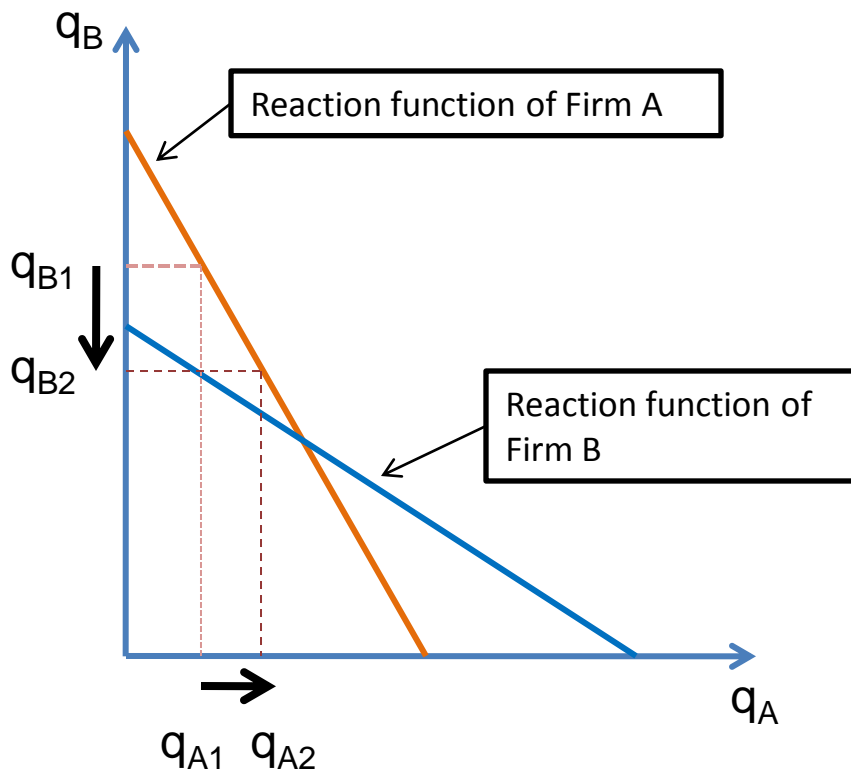
- Firm behaviour



- Reaction function



Cournot duopoly – Nash equilibrium



- Each firm's output depends on the output choice of the other firm: Nash strategy
- At the quantity levels defined by the intersection of the two reaction functions, neither firm has any incentive to change output: equilibrium

Algebra of Cournot duopoly - I

- Inverse demand curve

$$P = 1000 - 10Q$$

- Two identical firms

– Firm 1 produces q_1 and Firm 2 produces q_2

$$q_1 + q_2 = Q$$

- Cost structure

$$AC = MC = 50$$

Algebra of Cournot duopoly - II

- Profit maximising condition for a firm

$$MC = MR$$

- Decision

– How much to produce?

- Rewriting inverse demand curve

$$P = 1000 - 10(q_1 + q_2)$$

$$P = 1000 - 10q_1 - 10q_2$$

- Marginal revenue curve

$$\text{Firm 1: } (1000 - 10q_2) - 20q_1$$

$$\text{Firm 2: } (1000 - 10q_1) - 20q_2$$

Algebra of Cournot duopoly - III

- Profit maximisation

Firm 1: $(1000 - 10q_2) - 20q_1 = 50$

$$20q_1 + 10q_2 = 950$$

Reaction function of Firm 1

Firm 2: $(1000 - 10q_1) - 20q_2 = 50$

$$10q_1 + 20q_2 = 950$$

Reaction function of Firm 2

- Nash equilibrium

Solve the reaction functions simultaneously

$$20q_1 + 10q_2 = 950$$

$$10q_1 + 20q_2 = 950$$

Algebra of Cournot duopoly - IV

- Quantities in equilibrium

Solving the reaction functions simultaneously

$$q_1 = \quad , q_2 =$$

- Price in equilibrium

$$P = 1000 - 10(q_1 + q_2) =$$

- Profits in equilibrium

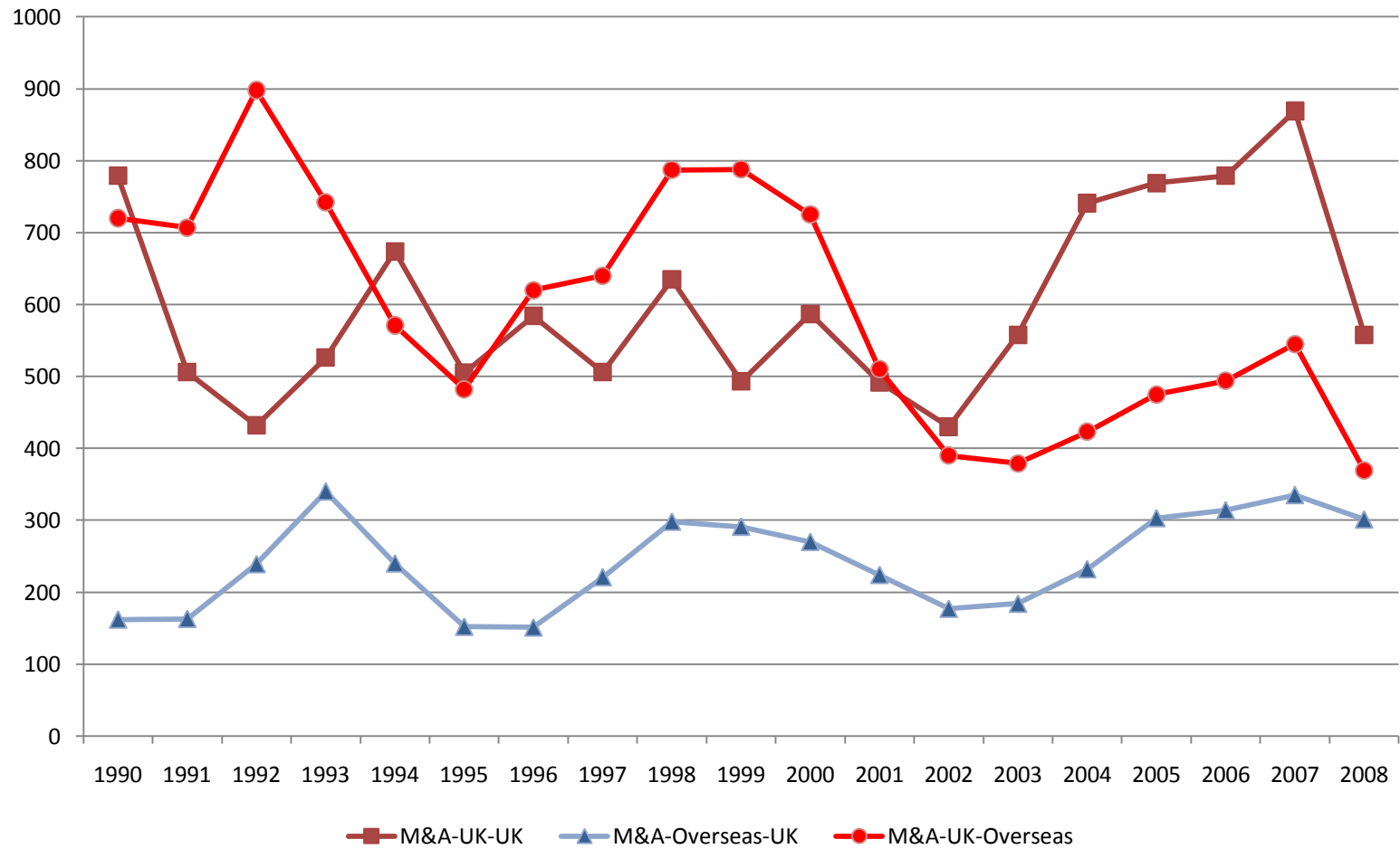
$$\pi_1 = (P - AC) \times q_1 =$$

$$\pi_2 = (P - AC) \times q_2 =$$

Strategy I – merger or collusion

- Market effectively has one multi-plant firm
 - Firm 1 has become Plant 1, and Firm 2 has become Plant 2
- Decisions
 - How much to produce?
 - How to distribute the output between the two plants?

Strategy I – incidence of merger



Source: Office of National Statistics

Strategy I – intuition

- The multi-plant firm will set output at the level where $MC = MR$
- It will allocate a larger share of the output to the firm with the lower cost
- If the plants have identical cost structures, the optimum output will be equally divided between the two plants

Algebra for Strategy I – I

- Inverse demand curve
 $P = 1000 - 10Q$
- Two identical plants
 - Plant 1 produces q_1 and Plant 2 produces q_2
 $q_1 = q_2 = Q/2$
- Cost structure
 $AC = MC = 50$
- Profit maximising condition for a firm
 $MC = MR$
 $50 = 1000 - 20Q$

Algebra for Strategy I – II

- Decisions

- How much to produce?

$$1000 - 20Q = 50$$

$$20Q = 950$$

$$Q = 47.5$$

- How to distribute output between the two plants?

$$q_1 = q_2 = Q/2 = 47.5/2 = 23.75$$

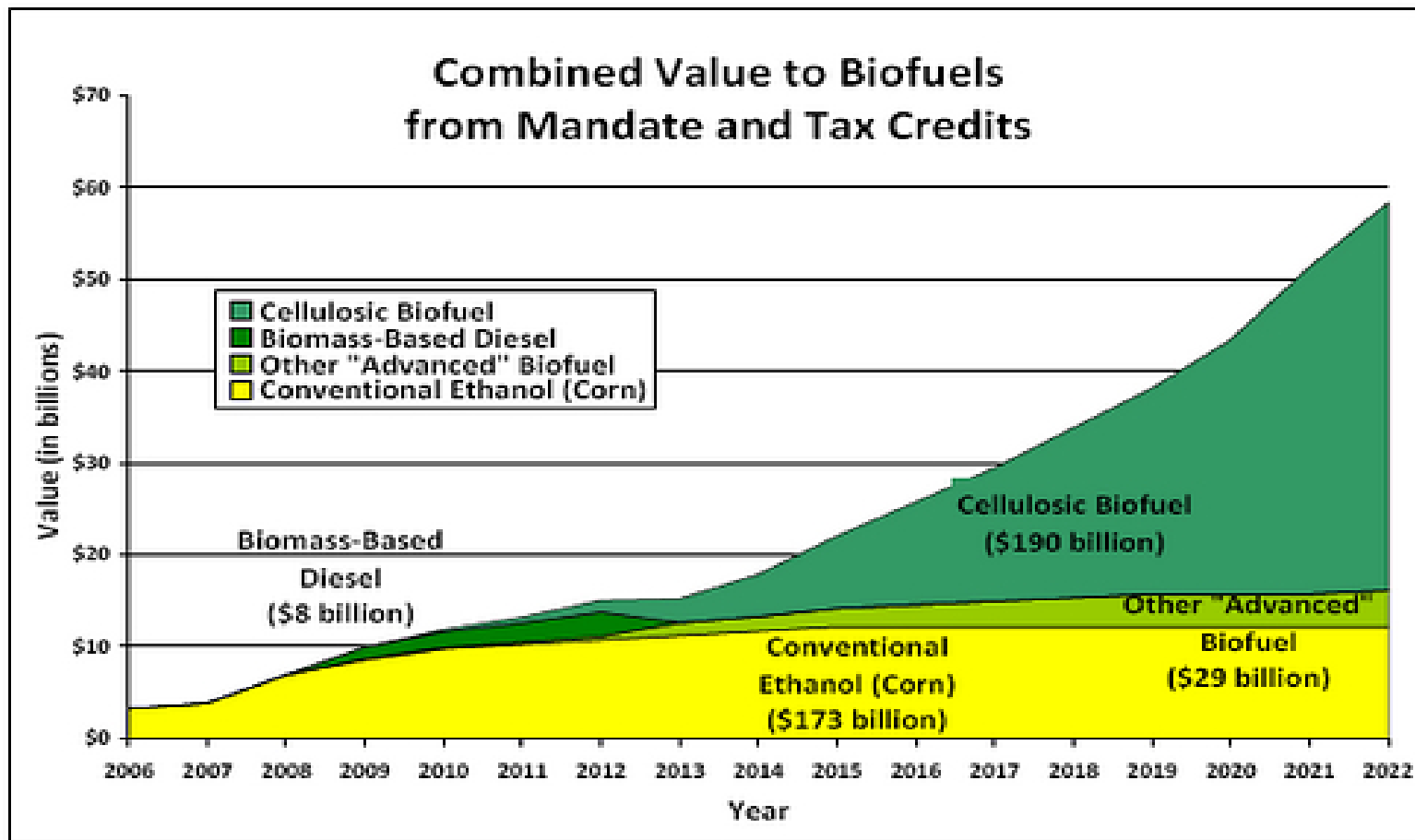
- Outcomes

$$P = 1000 - 10Q = 1000 - (10 \times 47.5) = 525$$

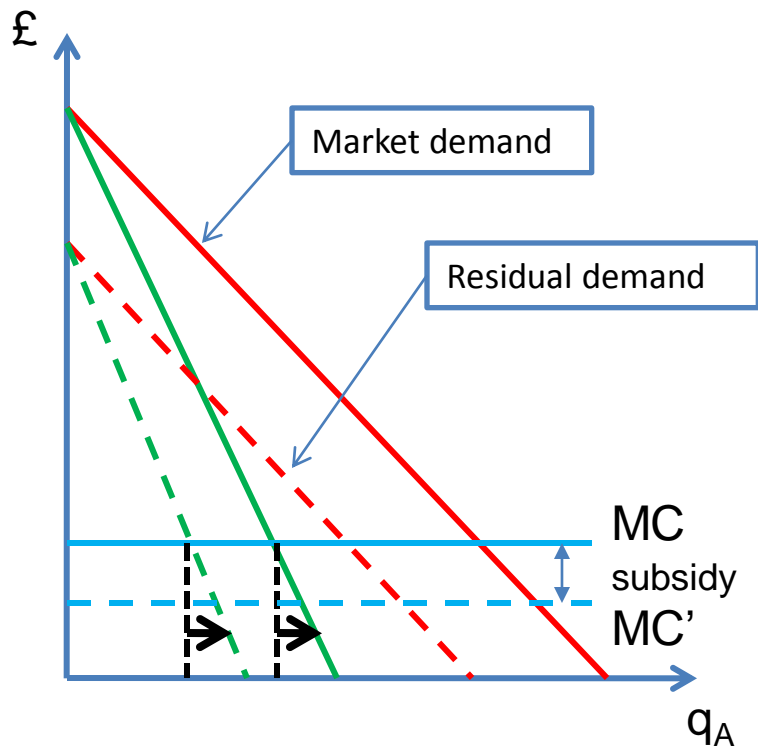
$$\pi = (P - AC) \times Q = (525 - 50) \times 47.5 =$$

In case of collusion, profit shared equally by the two firms

Strategy II – lobby for subsidy



Strategy II – impact of subsidy



- Subsidy reduces marginal cost of production
- The new marginal cost equals the marginal revenue at a higher output level
- The optimum output level of the firm is higher

Algebra for Strategy II – I

- Inverse demand curve
 $P = 1000 - 10Q$
- Two identical firms
 - Firm 1 produces q_1 and Firm 2 produces q_2
 $q_1 + q_2 = Q$
- Firm 1 gets a subsidy of 10 per unit of output
- Cost structure
Firm 1: $AC = MC = 50 - 10 = 40$
Firm 2: $AC = MC = 50$

Algebra of Strategy II - II

- Profit maximising condition for a firm

$$MC = MR$$

- Decision

– How much to produce?

- Rewriting inverse demand curve

$$P = 1000 - 10(q_1 + q_2)$$

$$P = 1000 - 10q_1 - 10q_2$$

- Marginal revenue curve

$$\text{Firm 1: } (1000 - 10q_2) - 20q_1$$

$$\text{Firm 2: } (1000 - 10q_1) - 20q_2$$

Algebra of Strategy II - III

- Profit maximisation

Firm 1: $(1000 - 10q_2) - 20q_1 = 40$

$$20q_1 + 10q_2 = 960$$

Reaction function of Firm 1

Firm 2: $(1000 - 10q_1) - 20q_2 = 50$

$$10q_1 + 20q_2 = 950$$

Reaction function of Firm 2

- Nash equilibrium

Solve the reaction functions simultaneously

$$20q_1 + 10q_2 = 960$$

$$10q_1 + 20q_2 = 950$$

Algebra of Strategy II - IV

- Quantities in equilibrium

Solving the reaction functions simultaneously

$$q_1 = \quad , q_2 =$$

- Price in equilibrium

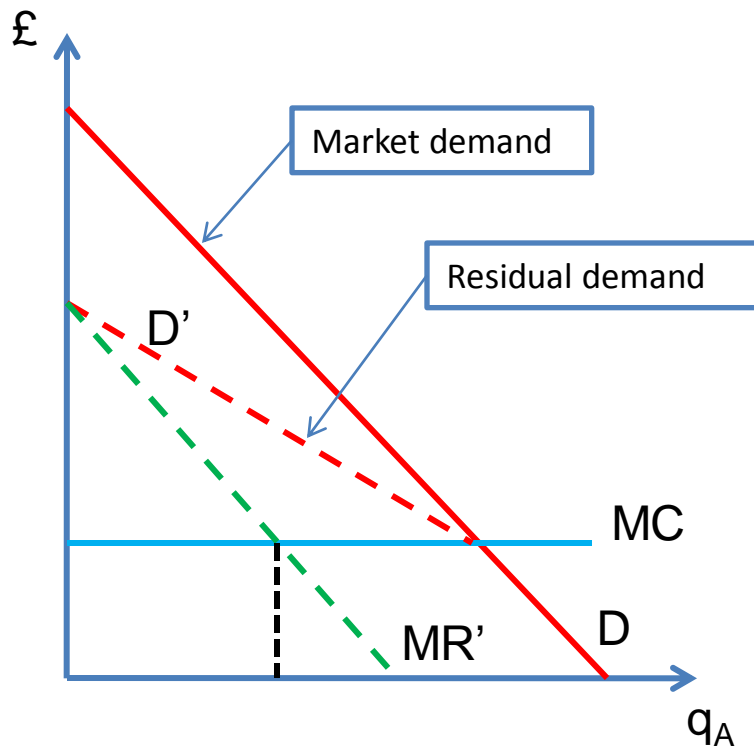
$$P = 1000 - 10(q_1 + q_2) =$$

- Profits in equilibrium

$$\pi_1 = (P - AC) \times q_1 =$$

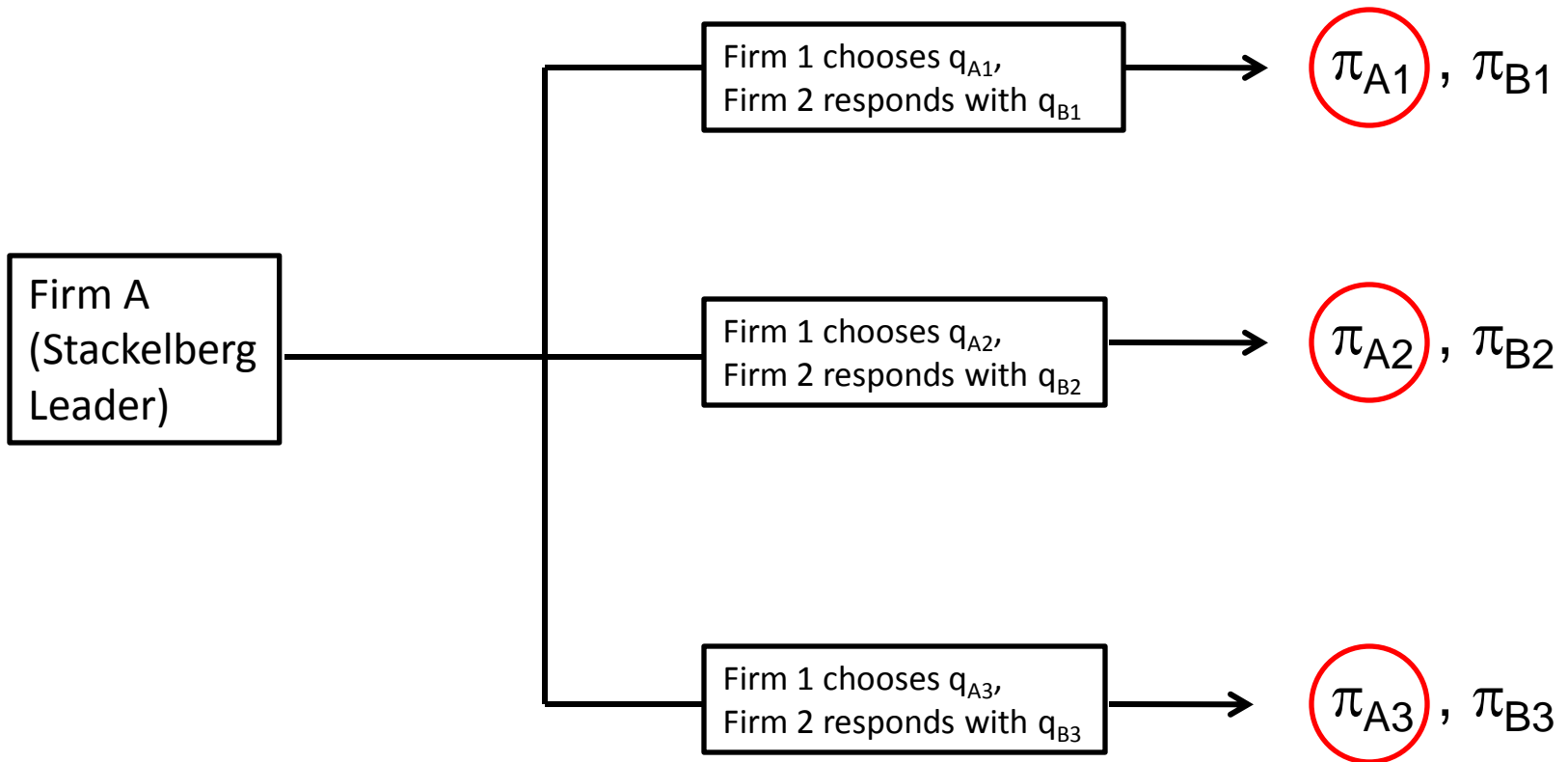
$$\pi_2 = (P - AC) \times q_2 =$$

Strategy III – become a Stackelberg leader



- Firm A (the Stackelberg leader) takes the strategic behaviour of Firm B into consideration
- Note the difference in the residual demand curve (relative to the Cournot competition scenario)
- In equilibrium, Firm A (the leader) would be better off and Firm B (the follower) would be worse off

Strategy III – become a Stackelberg leader



Algebra of Stackelberg duopoly - I

- Inverse demand curve
 $P = 1000 - 10Q$
- Two identical firms
 - Firm 1 produces q_1 and Firm 2 produces q_2
 $q_1 + q_2 = Q$
 - Firm 1 is Stackelberg leader
- Cost structure
 $AC = MC = 50$

Algebra of Stackelberg duopoly - II

- Profit maximisation of Firm 2

$$(1000 - 10q_1) - 20q_2 = 50 \quad (\text{from the algebra of Cournot})$$

$$10q_1 + 20q_2 = 950 \quad (\text{reaction function of Firm 2})$$

$$q_2 = (950 - 10q_1)/20$$

- Profit maximisation of Firm 1

1. Insert Firm 2's reaction function in Firm 1's demand curve

(i.e., substitute $q_2 = 47.5 - 0.5q_1$)

$$P = (1000 - 10q_2) - 10q_1 = 525 - 5q_1$$

2. Marginal revenue equals marginal cost

$$525 - 10q_1 = 50$$

Algebra of Stackelberg duopoly - III

- Quantities in equilibrium

First solve the profit maximisation problem of Firm 1

$$q_1 =$$

Then substitute q_1 into the reaction function of Firm 2

$$q_2 =$$

- Price in equilibrium

$$P = 1000 - 10(q_1 + q_2) =$$

- Profits in equilibrium

$$\pi_1 = (P - AC) \times q_1 =$$

$$\pi_2 = (P - AC) \times q_2 =$$