Reading map

As you must have noted, the lecture slides were based on a book by Rasmusen. However, the Waldman & Jensen text book also has a very good chapter on game theory.

Games – Example 1 (contd.)

Payoff matrix

		Others	
		Low	**High**
Saudi Arabia	**Low**	(10, 10)	(7, 12)
	High	(12, 7)	(9, 9)

- {*High*, *High*} is a dominant strategy equilibrium

The analysis proceeds as follows:

(a) If Saudi Arabia chooses *Low*, then the payoff for Others is 10 if they choose *Low* as well 12 if they choose *High*. Since 12 > 10, Others will chose *High*. If Saudi Arabia chooses *High* instead, the payoff for others is 7 if they choose *Low* and 9 if they choose *High*. Since 9 > 7, Others will choose *High*. In other words, Others will choose *High* irrespective of what Saudi Arabia's choice of action, i.e., choosing *High* is the dominant strategy for Others. (Note again the difference between action and strategy.)

(b) Now go down the column, and you should be able to demonstrate that *High* is also the dominant strategy for Saudi Arabia.

(c) This means that both Saudi Arabia and Others will choose *High*, i.e., {*High*, *High*} will be the (dominant strategy) equilibrium.

(d) In equilibrium, the payoff will be 9 each for Saudi Arabia and Others.

Games – Example 2 (contd.)

Payoff matrix

		Firm B	
		N	**S**
Firm A	**N**	(2, -2)	(2, -2)
	S	(1, -1)	(3, -3)

- It is a zero sum game
- {*N*} is a weakly dominant strategy for Firm B
- {*N*, *N*} is the equilibrium

To begin with notice that this is a zero sum game, i.e., for any combination of actions, the payoffs of the two players add up to zero. For example, when both Firm A and Firm B choose *N*, the payoff for Firm A is 2 and the payoff for Firm B is -2, such that the sum of the payoffs is zero. The implication is that in such games, when one player wins it is always at the expense of the other player.

In this game, if Firm A chooses *N*, then Firm B gets -2 if it chooses *N* as well and -2 once again if it chooses *S* instead. In other words, if Firm A chooses *N*, Firm B is indifferent between choosing *N* and *S*. If, on the other hand, Firm A chooses *S*, then Firm B clearly prefers *N* which gives it a higher payoff (-1) than *S* (-3). Hence, Firm B has a clear preference for *N* in one situation, and would not mind *N* in another. This makes choosing *N* a weakly dominant strategy of Firm B. If Firm B chooses *N*, of course, Firm A would choose *N* as well because *N* gives it a higher payoff (2) than *S* (1). In other words, the equilibrium is {*N*, *N*} and in the equilibrium the payoff is 2 for Firm A and -2 for Firm B.

Games – Example 3 (contd.)

Payoff matrix			
		Firm Y	
		VHS	Beta
Firm X	VHS	(2, 1)	(-1, -1)
	Beta	(-5, -5)	(1, 2)

- Neither firm has a dominant strategy
- {*VHS*, *VHS*} and {*Beta*, *Beta*} are the Nash equilibria
- First mover advantage matters

Try the same principle here. If Firm X chooses *VHS*, the payoff for Firm Y is 1 if it chooses *VHS* as well and -1 if it chooses *Beta* instead. Therefore, if Firm X chooses *VHS* then it would be best for Firm Y to choose *VHS*. If now Firm X chooses *Beta*, then the payoff for Firm Y is -5 if it chooses *VHS* and 2 if it chooses *Beta*. Hence, if Firm X chooses *Beta*, it would be best for Firm Y to choose *Beta* as well. Now we have a situation where the best choice of action of a player is not independent of what the other player chooses; it is dependent on the choice of the other player. Firm Y therefore does not have a dominant strategy. It has a Nash strategy which is as follows: *if Firm X chooses VHS then choose VHS as well, and if Firm X chooses Beta then choose Beta*. You can verify that Firm X has exactly the same Nash strategy. Hence, there are two Nash equilibria, namely, {*VHS*, *VHS*} and {*Beta*, *Beta*}.

Games – Example 4 (contd.)

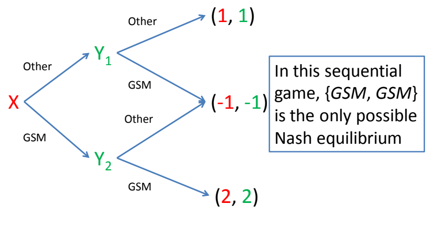
Payoff matrix			
		Firm Y	
		GSM	Other
Firm X	GSM	(2, 2)	(-1, -1)
	Other	(-1, -1)	(1, 1)

- Neither firm has a dominant strategy
- {*GSM*, *GSM*} and {*Other*, *Other*} are the Nash equilibria
- Can the firms communicate to ensure {*GSM*, *GSM*} equilibrium?

In this case, the players once again have Nash strategy. For example, Firm Y's strategy is as follows: *if Firm X chooses GSM then choose GSM, but if Firm X chooses Other then choose Other*. The two Nash equilibria are {*GSM*, *GSM*} and {*Other*, *Other*}. One of these two equilibria, namely, {*GSM*, *GSM*}, is clearly better than the other because in the {*GSM*, *GSM*} equilibrium both players get a payoff of 2, while in the {*Other* *Other*} equilibrium both players get a payoff of 1. Can the two firms choose *GSM* to get the higher payoff, without formally communicating with each other? Game theorists talk about focal points that make such coordinated action feasible. For example, if there is an independent review that says that *GSM* is a much better technology than *Other*, and both the firms read this review, then each might be inclined to choose *GSM* even without formally communicating with the other firm.

Games – Example 4 (contd.)

- Assumption: Firm X moves first



Thus far, we have assumed that both players choose their actions simultaneously. What if it is possible for one of the players to choose its action before the other player. Would this have an impact on the nature of the equilibrium? Consider once again the game discussed in the previous slide.

Now consider the situation where Firm X can choose its action first, and Firm Y has to follow. The game is then played out as follows: Firm X chooses either *GSM* or *Other*. Given the choice of Firm X, Firm Y can choose between *GSM* and

	Other as well. The combination of the two firm's actions gives us the payoffs. This is depicted by the game tree in this slide. The analysis proceeds as follows: (a) Firm X moves first and has to decide whether to choose *GSM* or *Other*. It knows that if it chooses *GSM* then Firm Y would choose *GSM* as well because that would give Firm Y a payoff of 2. If Firm Y chooses *Other* instead, it would get a payoff of -1. Similarly, if Firm X chooses *Other*, then it would be best for Firm Y to choose *Other* as well. (b) In other words, there would never be an equilibrium in which the payoffs are -1 each for Firm X and Firm Y (when one firm chooses *GSM* and the other firm chooses *Other*). If the only possible payoffs for Firm X are 1 (when both firms choose *Other*) and 2 (when both firms choose *GSM*), then it would make sense for Firm X to make the choice that would give it the higher payoff of 2, namely, *GSM*. (c) Unlike in the case where the two firms moved simultaneously, therefore, in this case only one equilibrium is possible, namely, {*GSM*, *GSM*}. This method of "solving" sequential move games is known as backward induction.
Repeated games • Finitely repeated • Infinitely repeated – Grim strategy • Choose Cooperate to start with • Continue to Cooperate until the other prisoner Cheats, and then choose Cheating forever – Tit-for-tat strategy • Choose Cooperate to start with • In each successive period, choose the strategy chosen by the other player in the previous period	Thus far, we have discussed one shot games, those that are played only once. What happens if games are repeated? If games are repeated finitely, nothing changes. Consider the prisoners' dilemma game discussed in class. Suppose that the game is repeated 5 times. Both prisoners would know that the 5th encounter is the last one, and hence during that encounter they would have every incentive to confess to the crime and incriminate each other. Prisoner A will then think that since Prisoner B would anyhow confess during the 5th round, he might as well confess during the 4th round and be one up on Prisoner B. Prisoner B, however, knows this, and hence he will want to confess during round 3, to be one up on Prisoner A. If you follow this line of argument you will see that it would then be inevitable for both prisoners to confess during the 1st round itself. If, however, the game is infinitely repeated, and prisoners have either grim strategy or tit-for-tat strategy then each prisoner would know that if he confesses and incriminates the other prisoner then the latter would always get an opportunity to extract revenge. In that case, both would be deterred from confessing and incriminating the

	other prisoner. NOTE: The nature of prisoners' dilemma was discussed in class. If you were not present in class and hence are not familiar with it, you will have to ask me about it in person.