Lecture 1
Games

"Game theory is concerned with the actions of individuals who are conscious that their actions affect each other."

Examples:

1. OPEC members deciding how much to produce every year

2. Chinese government purchasing steel from Arcelor Mittal

3. A large manufacturer of DVD players and a large manufacturer of DVD deciding on whether or not to adopt the Blu-ray standard

Essential elements:

a. players
b. actions
c. information
d. strategies
e. payoffs
f. outcomes
g. equilibria

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

Application to example #1:

Game: OPEC members deciding how much to produce every year.

Players: Saudi Arabia (S) and Others (O)

Definition:
"Nature" is a non-player who takes random actions at well defined points in time, with well defined probabilities.

Nature: World demand for oil (D) can be *Weak* or *Strong*, with probabilities of 0.7 and 0.3, respectively

Actions: Chosen by player *i* (a_i)

Definition:
Action set $A_i = \{a_i\}$ is the entire set of actions available to the player

An action is observable

Action set:

Both Saudi Arabia and Others can choose to produce either *High* (H) or *Low* (L), such that each player's action set is {L, H}

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

Definition:
Action combination is an ordered set of actions $a = \{a_i\}$, $\{i = 1, 2, \ldots, n\}$, each taken by a player in the game.

Order of play:

- Nature picks demand for oil, which can be *Weak* or *Strong*

- Saudi Arabia and Others simultaneously choose their 2007 outputs from their action sets {L, H}

Information:

Assumption:
Saudi Arabia knows whether the world demand for oil is weak or strong, but Others do not

Information sets:

Others: {D = *Weak*, D = *Strong*}
Saudi Arabia: {D = *Weak*} or {D = *Strong*}

Strategy:

A rule that tells a player what (s)he should do at any point in the game, *given his/her information set*

A strategy is unobservable

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

A plausible strategy for Saudi Arabia's:

$Q_S(D) = L$ if D = *Weak*
$= H$ if D = *Strong*

Related concepts:

- Strategy set
- Strategy combination

Payoff: Utility or expected utility or profits earned by a player as a consequence of actions chosen by him/her *as well as by other players*

Outcome:

Set of payoffs for the players once the game has played itself out

Equilibrium:

Combination of strategies chosen by the players to individually maximise their own payoffs

The same outcome may be associated with two different strategy combinations

Case 1:
The Golden Rule: *Low* output no matter what

Saudi Arabia: $Q_S = L$
Others: $Q_O = L$

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

Case 2:
An Eye for an Eye: Tit for tat

Saudi Arabia: $Q_S = L$ if $Q_O = L$
$Q_S = H$ if $Q_O = H$
Others: $Q_O = L$ if $Q_S = L$
$Q_O = H$ if $Q_S = H$

An outcome associated with both these strategy combinations is $\{Q_S = L, Q_O = L, D = \textit{Strong}, \pi_S(L, L), \pi_O(L, L)\}$

Prisoner's dilemma:

		Others	
		Low	High
Saudi Arabia	Low	(10, 10)	(7, 12)
	High	(12, 7)	(9, 9)

Payoffs to: (Saudi Arabia, Others)

Saudi Arabia = L, Others = H
Saudi Arabia = H, Others = H

Others = L Saudi Arabia = H
Others = H Saudi Arabia = H

It is a *dominant strategy* equilibrium

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

It is a *non-cooperative* game

Definition:
A cooperative game is one in which the players can make binding commitments

Iterated dominance:

Game:
- South Pacific during World War II (1943)
- Admiral Imamura has to transport soldiers across the Bismarck Sea
- He can take the shorter Northern route or the longer Southern route
- Admiral Kenney of the Allied forces has been asked to bomb the Japanese convoy
- He has to decide whether to send the bombers to look for the Japanese ships in the Northern route or in the Southern route

Players:
- Imamura
- Kenney

Action set:
{*North*, *South*}

Nature: ?

Order of play:

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

The players move simultaneously

Payoffs:

		Imamura	
		North	South
Kenney	North	(2, -2)	(2, -2)
	South	(1, -1)	(3, -3)

Payoffs to: (Kenney, Imamura)

It is a *zero sum game*

Kenney = North if Imamura = North
Kenney = South if Imamura = South

Imamura = North if Kenney = South
Imamura = South if Kenney = North

North is a weakly dominant strategy for Imamura

Definition:
A strategy is weakly dominant if it is a player's best response to any strategy of the other player, in the sense that the former's payoff from this strategy is no smaller than the payoffs associated with other strategy combinations, at greater than the payoffs associated with some strategy combinations

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

If Kenney realises that *North* is the weakly dominant strategy for Imamura, he will choose *North* as well

Changes in order of play:

If Kenney moved first, {*North*, *North*} would continue to be an equilibrium, but now {*North*, *South*} would be an equilibrium as well

Thought for the seminar:
What happens to Kenney's information set and his equilibrium strategy, and the equilibrium outcome if Imamura moves first?

Industrial application:

Two firms *A* and *B* are trying to maximise their respective market shares by choosing between the product designs *N* and *S*. Firm *A* has a marketing advantage and would like to compete with Firm *B* head-to-head. Firm *B*, however, would like to operate in a niche market.

<u>Nash equilibrium</u>:

Game:
- There are two pigs in a box, one large and one small
- Each can press a panel if it wants food; when the panel is pressed food is dispensed

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

- There is a disutility associated with pressing the panel
- The larger pig usually gets to eat a larger share of the dispensed food

Players:
- Large pig
- Small pig

Action set:

{*Press*, *Wait*}

Payoff:

		Small pig	
		Press	Wait
Large pig	Press	(5, 1)	(4, 4)
	Wait	(9, -1)	(0, 0)

Payoffs to: (Large pig, Small pig)

Equilibrium:

{*Press*, *Wait*} is an iterated dominant strategy equilibrium

Thought for the seminar:
Why?

{*Press*, *Wait*} is also a Nash equilibrium

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

Definition:
"A strategy combination s* is a Nash equilibrium if no player has the incentive to deviate from his strategy given that the other players do not deviate"

If the small pig chooses to *Wait* then it is best for the larger pig to *Press*, and if the larger pig *Press*es then it is best for the small pig to *Wait*

Things to note:

A Nash strategy is the best response to the other player's Nash strategy, *not to every strategy of the other player*

All dominant strategies are Nash strategies as well, *but not all Nash strategies are dominant strategies*

Payoff in a Nash puzzle:

		Brown	
		Left	Right
Smith	Up	(0, 1)	(-2, 0)
	Down	(0, -1)	(-1, 0)

Payoffs to: (Smith, Brown)

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

Equilibrium:

{*Up*, *Left*} and {*Down*, *Right*} are Nash equilibria

Why?

For Smith, the {*Up*} strategy is weakly dominated

Why?

But Smith would still prefer the {*Up*, *Left*} equilibrium which Pareto dominates {*Down*, *Right*}

Battle of the sexes:

Game:
- A man and a woman have to choose between a football game and a ballet
- The man dislikes ballet and the woman dislikes football
- They also prefer to be with each other

Payoff:

		Woman	
		Football	Ballet
Man	Football	(2, 1)	(-1, -1)
	Ballet	(-5, -5)	(1, 2)

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 1 & 2.

Payoffs to: (Man, Woman)

{*Football*, *Football*} and {*Ballet*, *Ballet*} are Nash equilibria

Each equilibrium is Pareto efficient

First-mover advantage matters

Industrial application:

Two competing firms want the same industry-wide standard for their products, but each wants a different standard, e.g., VHS and Beta