Lecture 2
Games …. Contd.

<u>Pure coordination</u>:

Game:
- Two mobile phone companies have to choose between GSM and an alternative protocol
- If they can choose the same protocol then each would sell more mobile handsets

Payoff:

		Firm Y	
		GSM	Other
Firm X	GSM	(2, 2)	(-1, -1)
	Other	(-1, -1)	(1, 1)

Payoffs to: (Firm X, Firm Y)

Equilibrium:

{*GSM*, *GSM*} and {*Other*, *Other*} are Nash equilibria

But {*GSM*, *GSM*} Pareto dominates {*Other*, *Other*}

What if it is not possible for the players to communicate before the game begins?

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 3 & 4.

Focal points

<u>The normal form</u>:

Outcome matrix:

It consists of all possible combinations of *actions* and the corresponding outcomes

Normal form:

It consists of all possible *strategy* combinations and the payoffs associated with these strategies

Example:

Pure coordination game with one difference: Company X moves first and Company Y follows with his response

Company Y's strategy set:

{*GSM*, *GSM*}
{*GSM*, *Other*}
{*Other*, *GSM*}
{*Other*, *Other*}

when

{*GSM*, *GSM*} can be interpreted as {If Company X chooses *GSM* then we choose *GSM*, if Company X chooses *Other* then we choose *GSM*}

Normal form of the game:

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="4">Company Y</td></tr>
<tr><td>{GSM, GSM}</td><td>{GSM, Other}</td><td>{Other, GSM}</td><td>{Other, Other}</td></tr>
<tr><td rowspan="2">Company X</td><td>GSM</td><td>(2, 2)</td><td>(2, 2)</td><td>(-1, -1)</td><td>(-1, -1)</td></tr>
<tr><td>Other</td><td>(-1, -1)</td><td>(1, 1)</td><td>(-1, -1)</td><td>(1, 1)</td></tr>
</table>

Equilibrium:

{*GSM*, {*GSM*, *GSM*}} Both choose GSM

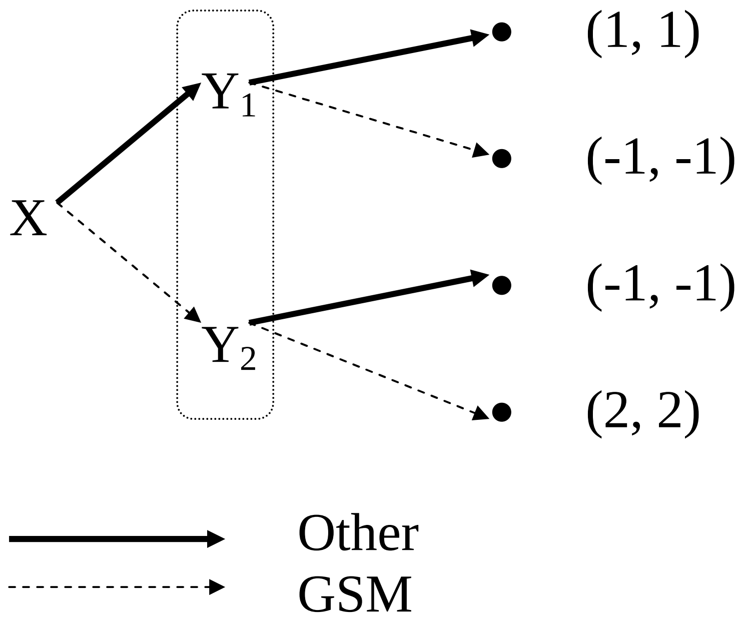
Thought for the seminar:
Similarly, find the other equilibria

<u>The extensive form</u>:

Elements:

- *Nodes*; each node has a predecessor and a successor
- *Branch*, which is an action from within an action set of a player, at a given node
- *Path*, which is a sequence of branches leading from a start node to an end node

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 3 & 4.



Payoffs to: (Company X, Company Y)

Once Company X moves, Company Y knows that it is either at node Y_1 or at Y_2, but does not know where it is

Company Y's information set *is not a singleton*

Definition:
A player's information set at any point of time in a game is the set of all nodes where the player can be, even though he is not sure about his exact location within that set

Based on: Rasmusen, Eric (1992) *Games and Information*, Oxford, UK and Cambridge, Mass.: Blackwell; Chapters 3 & 4.

Information:

Common knowledge:

> "Information is common knowledge if it is known to all players, each players knows that all of them know it, each of them know that all of them know that all of them know it, and so forth ad infinitum"

Information categories:

Perfect	Each information set is a singleton
Certain	Nature does not move after any player moves
Symmetric	No player has different information from other players when he moves, or at the end nodes
Complete	Nature does not move first, or her initial move is observed by each player

Rasmusen, Table 2.3

Thoughts for the seminar:
Classify the following poker rules:
- All cards are dealt face up
- All cards are dealt face down and a player cannot look at his own cards before he places a bet

- All cards are dealt face down and a player can look at his own cards
- All cards are dealt face up, but each player then secretly discards one card
- All cards are dealt face up, the players bet, and then each player gets another card face up
- All cards are dealt face down, and each player holds them against his forehead, such that everyone except himself can see them

<u>Mixed strategies</u>:

Payoff matrix for welfare game:

		Unemployed person	
		Search for work	Stay idle
Government	Dole	(3, 2)	(-1, 3)
	No dole	(-1, 1)	(0, 0)

Payoffs to: (Government, Unemployed person)

Equilibrium: There is no Nash equilibrium

Alternative: Rather than choose an action with certainty, each player can choose any of the available choices (in the action set) with a certain probability

Expected payoffs:

$$
\begin{aligned}
&E\pi_{govt} \\
&= P_a[3P_w + (-1)(1 - P_w)] \\
&\quad + [1 - P_a][(-1)P_w + 0(1 - P_w)] \\
&= P_a[3P_w - 1 + P_w] - P_w + P_aP_w \\
&= P_a[5P_w - 1] - P_w
\end{aligned}
$$

$$
\begin{aligned}
&E\pi_{unemp} \\
&= P_a[2P_w + 3(1 - P_w)] \\
&\quad + [1 - P_a][1P_w + 0(1 - P_w)] \\
&= 2P_aP_w + 3P_a - 3P_aP_w + P_w - P_aP_w \\
&= P_w[2P_a - 1] + 3P_a
\end{aligned}
$$

Optimal decisions:

Government:

$d\,E\pi_{govt}/dP_a = 0 \quad \Rightarrow \quad P_w{}^* = 1/5 = 0.2$

Unemployed person:

$d\,E\pi_{unemp}/dP_a = 0 \Rightarrow \quad P_a{}^* = 1/2 = 0.5$

Likely outcomes:

P{*Dole*, *Stay idle*} = 0.5(1 – 0.2) = 0.4

P{*No dole*, *Stay idle*} = 0.5(1 – 0.2) = 0.4

P{*Dole*, *Search for work*} = 0.5 × 0.2

P{*No dole*, *Search for work*} = 0.5 × 0.2

Interpretations:

(A) There are many unemployed people, 20% of whom would look for work while the rest remain idle

How an individual chooses between these actions/options is dependent on factors that are unrelated to the game

(B) Given the population of unemployed people, if the government chooses one individual randomly, there is a 20% chance that this person will search for a job

<u>Game of chicken</u>:

Payoff matrix:

		Brown	
		Stay	Swerve
Smith	Stay	(-3, -3)	(2, 0)
	Swerve	(0, 2)	(1, 1)

Payoffs to: (Smith, Brown)

Nash equilibria:

Two pure strategy Nash equilibria

{*Swerve*, *Stay*}

{*Stay*, *Swerve*}

Thought for the seminar:
Why are these Nash equilibria?

How would they decide who will *Stay* and who will *Swerve* if they do not communicate with each other before the game starts?

Mixed strategy equilibrium:

P{*Stay*} = θ

Brown's expected payoffs:

Eπ{*Swerve*} = θ (0) + (1 – θ) (1)
Eπ{*Stay*} = θ (-3) + (1 – θ) (2)

Thought for the seminar:
Why would these expected payoffs be the same for Smith?

From above we get
$1 - \theta = 2 - 5\theta$
$\theta^* = 0.25$

P{*Both survive*} = $1 - (\theta \times \theta) = 0.9375$

Subgame perfectness:

Problem:

Many games have multiple Nash equilibria, and one has to decide which of them is most sensible or likely

This problem is often associated with the normal form of the game which ignores who moves first

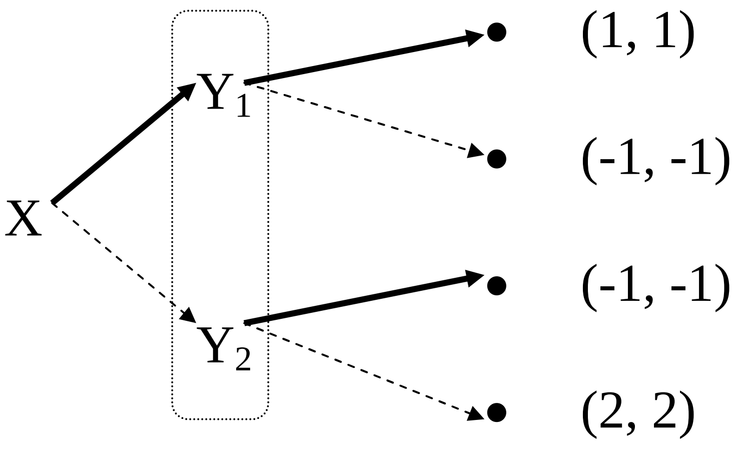
Game about choice of mobile technology:

Assumption:
Company X moves first

Equilibria:

{*GSM*, {*GSM*, *GSM*}}	Both pick *GSM*
{*GSM*, {*GSM*, *Other*}}	Both pick *GSM*
{*Other*, {*GSM*, *Other*}}	Both pick *Other*

Game tree:



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Other
GSM

Payoffs to: (Company X, Company Y)

Reviewing the equilibria:
If Company X deviates from the equilibrium path and chooses *GSM*, then Company Y would have no incentive to choose *Other*; it will choose *GSM* as well

If Company X knows this, then it will always choose *GSM*

{*Other*, {*Other*, *Other*}} would then not be an equilibrium; and {*Other*, *Other*} would be an irrational strategy for Company Y

The same logic works for {*GSM*, {*GSM*, *GSM*}} equilibrium and the {*GSM*, *GSM*} strategy for Company X

Thought for the seminar:
Why is {*GSM*, *GSM*} and inefficient strategy for Company Y?

{*GSM*, {*GSM*, *Other*}} is then an unique equilibrium

The other two are Nash equilibria but not *perfect* equilibria

Definition:
"A strategy combination is a perfect equilibrium if it remains an equilibrium on all possible paths, both the equilibrium path and other paths which branch off into different 'subgames'"

Subgame perfect equilibrium:

Definitions:
"A subgame is a game consisting of a node which is a singleton in every player's information partition, that node's successors, and payoffs at the associated end nodes"

"A strategy combination is a subgame perfect Nash equilibrium if (a) it is a Nash equilibrium for the entire game; and (b) its relevant action rules are a Nash equilibrium for every subgame"

Subgames in the extensive form of the Pure Coordination game:

(A) The entire game
(B) The subgame starting at node Y_1
(C) The subgame starting at node Y_2

Strategy {*GSM*, {*GSM*, *GSM*}} is Nash equilibrium in subgames A and C, but not in subgame B

Strategy {*Other*, {*Other*, *Other*}} is Nash equilibrium in subgames A and B, but not in subgame C

Strategy {*GSM*, {*GSM*, *Other*}} is Nash equilibrium in all three subgames

<u>Repeated games</u>:

Finitely repeated Prisoners' Dilemma:

In the last period of the game, in the absence of communication with the second prisoner, it would always be optimal for the first prisoner to *Fink* in the last round of the game

Given that they know this, it would be best for them to *Fink* in the second-last round of the game as well

Similarly, it would be best for them to *Fink* in all rounds of the game

Infinitely repeated game:

Grim strategy:

(a) Choose *Cooperate* to start with
(b) Continue to *Cooperate* until the other prisoner *Finks*, and then choose *Fink* forever

If the first prisoner chooses the *Grim Strategy*, it would be best for the second prisoner to choose *Grim Strategy* as well

It would be then best for both to continue to *Cooperate* forever

Tit-for-tat strategy:
(a) Choose *Cooperate* to start with
(b) In all successive period *n*, choose the strategy chosen by the other player in period *n-1*

If any prisoner chooses *Fink*¸ and both pursue *tit-for-tat*, the lifetime payoffs for both the players will be low than the {*Cooperate*, *Cooperate*} outcome

Hence, even if one prisoner chooses *Fink*, it would be best for the other to ignore it and to continue to choose *Cooperate*