OPTIONS & GREEKS

Study notes

1 Options

1.1 Basic information

An option results in the right (but not the obligation) to buy or sell an asset, at a predetermined price, and on or before a predetermined date. The predetermined price is known as the *strike price*, and the predetermined date is known as the *strike date*. A price, of course, has to be paid for this right, and hence an option has a price (which should not be confused with the strike price).

An option can be characterized by various attributes:

- If an option provides the right to buy an asset at a predetermined price, it is a *call* option. If it provides the right to sell an asset at a predetermined price, it is a *put* option.
- If an option can be exercised at any time until the strike date, it is called an *American* option, and if it can only be exercised on the strike date then it is called an *European* option.

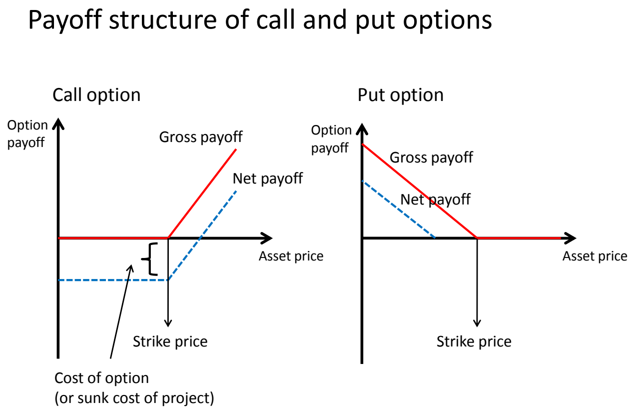
1.2 Payoffs from options

Consider first the call option, which confers the right to *buy* an asset at a predetermined price. Suppose that the strike price is $100. In that case, if the market price of the asset is less than or equal to $100, then there is no reason for the investor to exercise the option. If, however, the market price of the asset is greater than $100, then the investor would gain if he exercises the option. A numerical example can be used to provide further clarity:

- If the market price of the asset is $90, then the call option is *out of the money*, and it will not be exercised.
- If the market price of the asset is $100, then the call option is *at the money*, and it will not be exercised.

- If the market price of the asset is \$110, then the call option is *in the money*, and it will be exercised. Exercising the option will result in a payoff of \$10 (= \$110 - \$100).

Figure 1: Option payoff



The payoff structure discussed above can be summarized graphically; see Figure 1. The graph to the left summarizes the payoff for a call option. We have payoff from the option along the vertical axis, and the price of asset alone the horizontal axis. The gross payoff from the option is given by the red line. As we have seen above, the call option will not be exercised so long as the market price of the asset is less than or equal to the strike price. When the option is not exercised, the payoff from it is zero, there is neither a loss nor a gain. Once the market price of the asset exceeds the strike price then the option is exercised, and there is always a positive gross payoff that increases with the market price of the asset. For example, if the market price of the asset is \$110, and the strike price is \$100, then the payoff is \$10. Similarly, if the market price is \$120, then the payoff is \$20, and so on. Hence, the red gross payoff line moves along the horizontal axis with value zero until the strike price, and after that it rises steadily with the market price of the asset.

The blue line reflects the net payoff, i.e., the difference between the gross payoff from an option and the price at which the option has to be purchased. The net payoff from an option is necessarily less than the gross payoff. It is easy to see from the graph that if an investor purchases a call option, then there is automatically a floor for his downside risk; the maximum he can lose is the price that he pays for the option. On the other hand, if the market price of the asset rises above the strike price, there is no limit to the gains that he can make.[1]

In Figure 1, the graph to the right highlights the payoff for a put option, which confers to its owner the right to sell an asset at a predetermined price. Suppose that the strike price once again is $100. In that case, if the market price of the asset is greater than or equal to $100, then there is no reason for the investor to exercise the option. If, however, the market price of the asset is less than $100, then the investor would gain if he exercises the option. A numerical example can be used to provide further clarity:

- If the market price of the asset is $90, then the put option is *in the money*, and it will be exercised. Exercising the option will result in a payoff of $10 (= $110 - $100).
- If the market price of the asset is $100, then the put option is *at the money*, and it will not be exercised.
- If the market price of the asset is $110, then the put option is *out of the money*, and it will not be exercised.

Once again, the red line represents the gross payoff and the blue line represents the net payoff. As the market price of the asset increases, the gross payoff (i.e., the benefit from owning the right to sell the asset at the strike price) declines. Once the market price equals and then exceeds the strike price, the owner of the put option is better off not exercising the option and selling the asset at the market price instead. Hence, for market prices in excess of the strike price, the gross payoff is zero. Since a price has to be paid to own the put option, the net payoff is less than the gross payoff.

2 Option price

Option price is computed using the Black-Scholes formula. The formula indicates that this price depends on the following:

[1] The opposite is true for the person who has written or sold the call option. So long as the market price of the asset is below the strike price, the option is not exercised, and hence his net benefit is the cash that he raised by selling the options. But once the market price of the asset exceeds the strike price, the seller starts losing money. The loss may be actual, if he has to buy the asset for (say) $110 from the market and sell it to the owner of the call option for $100 (the strike price). The loss may also be opportunity lost, if the seller of the option writes a *covered call* whereby the call option sold is backed by actual ownership of the asset. In that case, when the market price of the asset exceeds the strike price, instead of earning $110 from its sale, the seller of the call option can at best earn $100 by selling the asset to the owner of the option, at the strike price.

- *Interest rate*: Consider a situation where Person A has written a put option and has sold it to Person B for \$1. Let the interest rate be 10%, and let the strike date for this option is 1 year down the road. Since Person A has already received \$1, he can keep it in the bank at 10%, and receive \$1.10 after a year. Now, in order for the option to be fair, both Person A and Person B should have the same expected payoff after 1 year. Hence, Person B should also expect to receive \$1.10 after a year. What will happen now if the interest rate increases to 15%? If the option price does not change, such that Person A continues to receive \$1 for the put option that he has written, then he will receive \$1.15 after 1 year. To maintain fairness, therefore, Person B will have to expect a faster rise in the market price of the underlying asset. If, on the other hand, there is no change in the expected market price of the asset in a year's time, such that Person B continues to expect a net payoff of \$1.15, then the price of the option would have to be $\frac{1}{1+0.15}$, which is \$0.869. In other words, interest rates affect option price.

- *Volatility*: Asset prices change over time. In some cases, they move in only one direction. For example, the value of Greek sovereign debt has steadily declined over some weeks and months. In some other cases, however, asset prices move up and down, just as stock prices around the world have been moving up and down sharply over the past few weeks. The sharper the change in an asset's price, the greater is the volatility associated with the price of this asset. Suppose that an asset is priced at \$100, and historical estimates of volatility suggest that it's price can go up or down by 10% over a one year period. For the owner of a put option with a strike price of \$100, there is no downside risk associated with an increase in the market price of the asset beyond the strike price. At the same time, if the market price is below the strike price of \$100, he will earn a positive payoff. Given the estimated volatility of 10%, the maximum that this investor can expect to earn is \$10.[2] It is easily seen than if the volatility of the asset price increases to 20%, then the maximum amount this investor can expect to earn is \$20. Since his expected payoff would be higher, he should therefore pay more to buy this put option. In other words, the price of the option is affected by the volatility of the market price of the asset.

[2]The volatility suggests that the lowest expected price is 10% below \$100, i.e., \$90. Given the strike price of \$100 for the put option, this would result in a payoff of \$10 to the owner of the option.

- *Dividend*: It is obvious that this factor affects only prices of shares; dividends are generally not associated with other kinds of assets. Dividend payouts affect share price. If a dividend payout in the future results in a decline in share price[3] then the expected payoff of both the owner of a put option and the owner of a call option will change; the former will expect to gain more while the latter will expect to gain less. In either case, there should be a change in the price of the option to reflect the change in expected payoff of the option owners. In other words, option prices (for shares) are affected by dividend policies of companies.

- *Time to expiration*: The longer the time to expiration of an option, the greater is the possibility that the option would be in the money before it expires. Hence, options with longer time to maturity are priced higher than those with shorter time to maturity.

3 Greeks

3.1 Delta

An option's **delta** is the rate of change of its price with respect to the change in market price of the underlying asset, when everything else is unchanged. For example, if a 1% change in the price of the asset results in a 0.5% change in the price of the option, then the delta of the option is 0.5.

As we have already seen, if the market price of the underlying asset increases, the payoff increases for a call option either remains unchanged (if the market price continues to be below the strike price) or it increases (if the market price exceeds the strike price). Since the price of an option depends on the payoff it generates for the investor, any change in the market price of the asset should therefore result in either a zero change or a positive change to the price of a call option. Conversely, for a put option, the payoff either decreases as the market price of the underlying asset increases, or it remains unchanged. Hence, for a given change in the market price of the asset, the change in the price of a put option should either be zero or negative.

Now consider an in the money call option. It gains value from two sources:

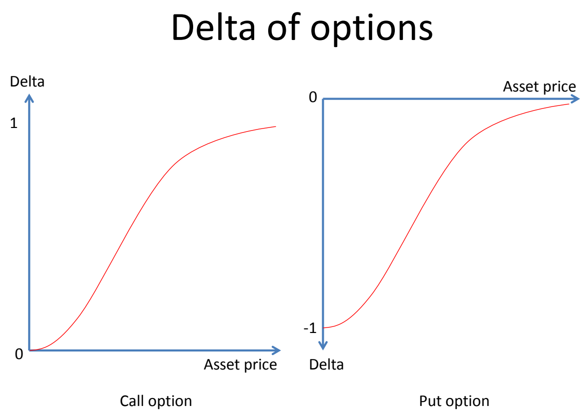
[3] Share prices reflect the cash flows of companies. When dividend is paid out, a company loses cash, and hence the price of the share falls. Specifically, if the dividend paid out is \$1 per share, then the price per share drops by \$1 as well.

- *Intrinsic value*: If the market price of the underlying asset increases, and the option is American in nature, the investor can immediately exercise it to cash in on the price increase. This is the intrinsic value of the option.

- *Option value*: The call option ensures that the downside risk is limited – there is a limit to the loss the investor will suffer on account of changes in the market price of the underlying asset, while the upside potential is limitless. The option value is associated with the limit to the downside risk to the investor who owns the option.

Suppose that the market price of the underlying asset increases by \$1. In that case, the intrinsic value of an in the money (American) call option increases by \$1 as well. However, since the market price of the asset already exceeds the strike price and has gone up further, there is a lower probability that this price would be below the strike price on or below the strike date. Hence, the option value of the call option decreases. The net impact of the \$1 change in the market price of the call option, therefore, is less than \$1, and so the upper limit of a call option's delta is 1. Similarly, the lower limit of a put option's delta is -1.

Figure 2: Delta



The bounds of the delta for call and put options can therefore be summarised as follows:

$$0 \leq \text{delta of a call option} \leq 1$$
$$0 \geq \text{delta of a put option} \geq \text{-1}$$

Both call and put options have delta of 0 when they are deep out of the money. When they are deep in the money, a call option has a delta of 1, and a put option has a delta of -1. This is highlighted in Figure 2.

3.1.1 Delta hedging

Consider a portfolio of 100 call options with a delta (δ) of 0.5. If, therefore, the market price of the underlying asset falls by $1 per unit, the value of each option would fall by $0.50. The overall decline in the value of the portfolio would be $50. Suppose that the investor who owns this portfolio shorts 50 units of the underlying asset. This means that he sells the asset at the current price without actually owing it, buys it from the market when the price declines by $1, and delivers it to the buyer. It is easy to see that this transaction will result in a profit of $1 per unit of the asset, i.e., $50 overall. In other words, the $50 loss in the value of the call options is completely offset by shorting 50 units of the underlying asset, when 50 equals δ times 100, the number of call options in the portfolio. The portfolio then is perfectly hedged.

Consider now a portfolio of 100 put options with a delta of -0.5. If the market price of the underlying asset falls by $1 per unit, the value of the options portfolio will rise by $50. If the investor wants to perfectly hedge this portfolio, he will have to buy 50 units of the underlying asset. The value of the asset will decline by $50 when the price per unit falls by $1, and this will completely offset the rise in the value of the options portfolio.

To summarise, an investor will have to adopt the following strategies to make their portfolios *delta neutral*:

$$n \text{ call options} + \text{short } (\delta \times n) \text{ units of the underlying asset}$$
$$n \text{ put options} + \text{buy } (\delta \times n) \text{ units of the underlying asset}$$

3.2 Gamma

An option's **gamma** is the change of its delta with respect to the change in the market price of the underlying asset. If the gamma is small, then the delta changes very little as the price of the underlying asset changes. If, on the other hand, the gamma is large, then the delta changes significantly with the price of the underlying asset.

It is obvious that this has implications for hedging strategies. We have already seen that the delta of an option (and hence an options portfolio) has to be taken into consideration to hedge the options portfolio. If this delta itself changes as the market price of the underlying asset price changes, an investor with an options portfolio who wants to maintain a delta neutral portfolio at all times will have to keep adjusting his portfolio of the underlying assets. This process is known as *dynamic hedging.*

The gamma of an option is large if the option is at the money and it is close to expiration. For such an option, even a small change in the market price of the underlying asset can result in the option expiring in the money. Hence, the change in its delta on account of a change in the market price of the asset (i.e., its gamma) is large.

3.3 Rho

An option's **rho** is the rate of change of its value with respect to a change in the interest rate. For example, if the rho is -50, then a 100 basis points increase in the risk free interest rate reduces the value of the option by 0.50.

3.4 Vega

An option's **vega** is the change of its value with respect to the volatility in the price of the underlying asset. For an European option, the vega of a call and a put option are the same. Vega is also positive for both call and put options; an option becomes more valuable when the volatility of the market price of the underlying asset increases. It is easy to see that, other things being the same, the vega of an option is lower when it is close to expiration. Vega is also higher for options when the market price of the underlying asset is close to the strike price.

3.5 Theta

An option's **theta** is the rate of change of an option's price with respect to time, when everything else is unchanged. It is also known as the *time decay* of the option. For example, if an option has a theta of -25, it means that if 0.01 years (approximately 2.5 trading days) pass without a change in the market price and volatility of the underlying asset, then the value of the option will decline by 0.25. An option's theta is generally negative; an option becomes less valuable as the expiration date approaches.