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## **Directional Mobility of Ratings**

*By: Sumon K Bhaumik and John S. Landon-Lane*

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# Directional mobility of debt ratings\*

Sumon Kumar Bhaumik<sup>†</sup>      John S. Landon-Lane<sup>‡</sup>  
Economics and Finance      Dept. of Economics  
School of Social Sciences      Rutgers University  
Brunel University

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## Abstract

In this paper we describe a method to decompose a well-known measure of debt ratings mobility into its directional components. We show, using sovereign debt ratings as an example, that this directional decomposition allows us to better understand the underlying characteristics of debt ratings migration and, for the case of the data set used, that the standard Markov chain model is not homogeneous in either the time or cross-sectional dimensions. We find that the directional decomposition also allows us to sign the change in quality of debt over time and across sub-groups of the population.

Keywords: Ratings migration, Mobility, Sovereign debt

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<sup>†</sup>Address: Brunel University, Economics and Finance, School of Social Sciences, Marie Jahoda, Uxbridge UB8 3PH, UK. Email: Sumon.Bhaumik@brunel.ac.uk

<sup>‡</sup>Corresponding author. Address: 75 Hamilton St, New Brunswick, NJ 08812, USA. Email: lane@econ.rutgers.edu

# 1 Introduction

Arguably, the widespread adoption of Basle norms for supervision of banks and the rapid growth of the market for credit derivatives are among the two most important developments in the world of banking and finance since the abandonment of the Bretton Woods system in 1973. The Basle norms, which came into force in nine of the G-10 countries in 1992, and have since been adopted by bank regulators in a wide range of countries, initially penalised banks for risk associated with their credit portfolios, by requiring them to maintain a minimum amount of capital in proportion to the risk weighted assets on their balance sheets. Basle II recognised the need to take into consideration market risk and organizational risk as well, in the process of building a sound banking system. Banks that are subjected to Basle II regulations are required to undertake value-at-risk (VaR) exercises to determine the extent of the market risk of their asset portfolio. Over roughly the same time period, making a quantum leap from a nascent market up until the middle of the nineties, the size of the credit derivatives market exceeded USD 8 trillion at the end of 2006. Credit default swaps accounted for roughly 50 percent of the market. Altman (1998) provides an excellent discussion about the importance of understanding the patterns of credit rating migration.

It is easily seen that the common thread linking the Basle norms for banking regulation and the rapidly growing market for credit derivatives is that both attach significant importance to unfavorable events in the market. Changes in interest and exchange rates, as well as equity and commodity prices can adversely affect the value of a bank's asset portfolio, and Basle II aims to ensure, among other things, that the capital base of a bank would be able to absorb an adverse movement in these market prices without resorting to bail out and closure. The contracts exchanged in the market for credit derivatives, on the other hand, hinge on events that could either be defaults on a loan or those that are close approximations of a default, e.g., postponement of payment of interest. The likelihood of the occurrence of an unfavorable event that can reduce the value of an asset portfolio

or trigger an event included in a credit derivatives contract are, in turn, related to the phenomenon of ratings migration. Banks and investors have to take into consideration the probability of ratings downgrades (or, more generally, changes) of securities (or their issuers) that are either directly included in their portfolios or are underlying assets for credit derivatives products of which they are a counter-party. Specifically, they have to factor in the likelihood of ratings downgrades (and upgrades) when they decide on the prices of these securities and derivatives products, as also the likely future needs for capital (in the case of a bank).

While there are several ways to model the likelihood of a ratings migration, most of these models make use of assumptions that are unrealistic (Albanese & Chen 2006). For example, Jarrow et al. (1997) postulate that the likelihood of an upgrade and a downgrade are the same even though it can be convincingly argued, for example, that the likelihood of a sovereign rating downgrade is often higher for developing countries while that of an upgrade is higher for industrialized (or rapidly industrializing) countries. More importantly, they compute a composite likelihood of ratings migration that is not informative about the individual probabilities of a downgrade and an upgrade. Yet, as we have argued above, measures of these individual probabilities are important both to compute an accurate VaR measure for an asset portfolio and to accurately price a derivatives product that is structured to protect against a movement in one direction, namely, a default. In this paper, using sovereign ratings data obtained from Moody's Investors Service, for the 1996-2005 period, we address this relatively unexplored methodological aspect of modeling ratings migration.

In this paper we use a time-homogeneous discrete-state first-order Markov model to estimate credit migration (transition) matrices for sovereign debt ratings of various groups of countries. We are interested in testing for differences in the inferred migration matrices across different groups of countries and across different economic conditions. As in Jafry & Schuermann (2004) we argue that the standard metrics that are used to

distinguish migration matrices (for example, the mobility indices introduced by Shorrocks (1978) that are based on the eigenvalues of the migration matrix) do not fully describe the important characteristics of credit rating migration. Jafry & Schuermann (2004) argues that an important characteristic of ratings migration is the size of the jump, i.e., a movement of two ratings classes in one period is different to a movement of one ratings class in the same period. However, like the mobility measures of Shorrocks (1978), the mobility index suggested by Jafry & Schuermann (2004) does not distinguish between upward movements and downward movements in the ratings distribution. In this paper we use the directional mobility measures introduced in Gang et al. (2004) to test for differences in two migration matrices based on their implied directional mobility thereby allowing us to fully characterize the directional mobility of sovereign debt. We therefore get a better understanding of the underlying dynamics of sovereign debt migration. In addition, we are able to estimate directional mobility scores conditional on the initial ratings class of the bonds. It is evident that these conditional measures of upward and downward mobility of ratings have significant implications for two important sets of investors, namely, those who invest in “cross-over” bonds and those that invest in high yield bonds.

Bayesian methods are utilized in this paper which allow us to generate exact finite sample tests of differences in sovereign debt ratings migrations. The choice of sovereign ratings data and the aforementioned time period enriches our analysis in several ways. First, since the early nineties, a large number of emerging markets (and corporate entities therein, whose ratings are usually capped at the corresponding sovereign ratings) have regularly accessed the global financial market to raise funds. Our sample, therefore, includes a number of emerging markets from Asia, Central and Eastern Europe, and Latin America, thereby allowing us to compare and contrast not only the likelihood of upward and downward mobility of ratings of industrialized and emerging economies, but also those of emerging economies belonging to different regions of the world. Second, the

time period of our data includes three clearly identifiable adverse shocks that presumably had global implications, namely, the Asian crisis of 1997, the Russian default of 1998 and the Argentinean default of 2001. We are, therefore, able to identify the impact of these crises on the probabilities of upgrades and downgrades for each of the directly affected regions, the corresponding probabilities of other regions of emerging markets, and the probabilities associated with the ratings of the developed countries that were lenders to and investors in these regions. It is easily seen that our data allows us to examine both the impact of country-specific or regional events on the likelihood of ratings upgrades and downgrades, and the nature and pattern of ratings contagions. In other words, it offers us a scope to comprehensively demonstrate the advantages of our methodology.

We find that the time homogenous assumption is rejected for our sample and our time period. We also show that the ratings migration is not homogenous in the cross-sectional dimension as well. As such, we are able to demonstrate that knowing the directional mobility of sovereign debt ratings is important in fully understanding the underlying dynamics of the debt ratings migration. In some cases we find that the directional mobility scores allow us to conclude that the ratings migration matrix has changed between sub-periods where, otherwise, using only the standard mobility measures we would have concluded that there was no difference. We also show that the directional mobility scores allow us to better explain the differences between different sub-groups of countries and between different time periods. This, in turn, enables us to discuss the relative change in the quality of the underlying debt directly from the directional mobility scores that we could not do using the standard overall mobility measures.

The rest of the paper is structured as follows: Section 2 describes the model and the estimation method and the directional mobility measures used in this paper to distinguish the ratings migration matrices. Section 3 describes the data and the prior distributions used in the analysis while Section 4 describes the results. We use these results to demonstrate the importance of separately estimating upward and downward mobility scores

for ratings migration and, correspondingly, the shortcoming of an overall mobility score. Finally Section 5 concludes.

## 2 Method

### 2.1 Brief Review of the Methodological Literature

The dynamics, and in particular the mobility, of sovereign debt ratings is studied in this paper using a first order Markov chain. The use of Markov-chain models to study mobility has a long history with notable early contributions by Champernowne (1953) and Prais (1955). More recently Shorrocks (1976, 1978) discussed the Markov assumption with reference to measuring income mobility and introduced measures of mobility that were functions of the estimated migration (transition) matrices.

A number of papers have also applied Markov models to studying credit rating migration in the literature. These papers have concentrated on a number of issues, and, in our paper, we have addressed all the methodological concerns raised in the course of earlier research. To begin with, there is a discussion in the literature about whether the time-homogeneity assumption is valid for the case of bond rating migration. Authors such as Bangia et al. (2002), Nickell et al. (2000) and Wei (2003) argue that we should condition on macroeconomic factors such as the business cycle when estimating credit migration matrices, that the migration matrices are sensitive to the underlying economic conditions. In the empirical part of this paper, we control for changing macroeconomic conditions by breaking our sample into three sub-samples, namely, a period of the Asian and Russian crises of the late 1990's, a period of recovery in these regions of the world and simultaneously a crisis in Latin America, and finally a period free of crises and yet one fraught with uncertainty about rising energy prices and sustainability of growth in the United States.

Other approaches to relaxing the time homogeneity of the Markov model include

Frydman & Kadam (2004) which takes into account the age of the bond. They argue the relatively young bonds face different probabilities of migration than older bonds and show that a model that takes this into account yields statistically and economically different estimates of credit migration probabilities than the standard time-homogeneous first order discrete state Markov model. However, their results also show that for bonds that have been in existence for longer than four years, the estimates of the ratings migration probabilities for their approach is almost identical to those estimated from the standard Markov model. Given our data are on sovereign bonds that have been rated for many years prior to 1996, the first year in our sample period, we think that the results we report in this paper do not suffer from the problem discussed in Frydman & Kadam (2004).

Frydman & Schuermann (2004) relax the homogeneity assumption by estimating a random mixture Markov model where the probability of transition is modeled by two credit migration matrices. Each bond's ratings migration probability has a positive probability of being described by each of the two migration matrices. They show that this random mixture model statistically dominates the standard model for corporate bonds. In keeping with the spirit of this line of reasoning, in this paper, we account for the possibility that different bonds could face different migration probabilities by separating the sovereign bonds into sub-groups based on country characteristics.

Finally, recent work by Fuertes & Kalotychou (2007) show that for sovereign debt, downgrades and upgrades should be treated differently. As mentioned earlier, this is the focus of our paper. We estimate a discrete-state first order Markov model with the aim of testing for differences in upward and downward ratings mobility for different groups of countries during different time-periods of recent history.

## **2.2 Markov Chains and Ratings Migration**

One of the most appealing aspects of using a Markov-chain to model ratings dynamics across individual countries is the ability to investigate issues such as differences in ratings

mobility over time, among subgroups of the population. The Markov assumption is a natural way of thinking about ratings dynamics while imposing only minimal theoretical structure on the dynamics of the system.

The first order discrete-state Markov model is as follows: Let there be  $C$  ratings classifications where  $C$  is a finite number. Let  $\pi_t = (\pi_{1t}, \dots, \pi_{Ct})'$  be the distribution across the  $C$  classes where  $\pi_{kt}$  is the proportion of the total population that is in class  $k$  at time  $t$ . Therefore the variable  $\pi_t$  defines the “state” of the world at time  $t$ . The first-order Markov assumption implies that the state of the world today is only dependent on  $\pi_{t-1}$ . That is,

$$P(\pi_t | \pi_{t-1}, \pi_{t-2}, \dots, \pi_{t-j}) = P(\pi_t | \pi_{t-1}) \quad \forall j = 2, 3, \dots, \quad (1)$$

where  $P(\cdot)$  represents the conditional probability distribution of  $\pi$ . Define the probability of transiting (migrating) from class  $i$  in period  $t-1$  to class  $j$  in period  $t$  to be  $P(\pi_t = j | \pi_{t-1} = i) \equiv p_{ij}$  so that the Markov transition (migration) matrix,  $\mathbf{P}$ , can be defined as  $\mathbf{P} = [p_{ij}]$ . Then the first order Markov chain model is

$$\pi_t' = \pi_{t-1}' \mathbf{P}. \quad (2)$$

The initial income distribution is  $\pi_0$  and it is simple to show that  $\pi_t' = \pi_0' \mathbf{P}^t$ .

This paper uses Bayesian methods to estimate and make inferences from the Markov chain model outlined above. One important consequence of using Bayesian methods is that it is simple to characterize the exact finite sample properties of the distribution of any function of the primal parameters,  $\pi_0$  and  $\mathbf{P}$ , of the model. For example, we are able to characterize the distribution of various mobility indices such as the probability of moving to a higher income class. More detail about the particular mobility indices that we are interested in can be found in Section 2.3 below.

Before discussing in detail the measure of mobility and the tests used in this paper

we first discuss our sampling scheme. We observe  $N$  countries over  $T$  time periods and place them into  $C$  classifications. Let  $i \in \{1, 2, \dots, C\}$ ,  $n \in \{1, 2, \dots, N\}$ , and let  $t \in \{1, 2, \dots, T\}$ . For each country,  $n$ , define

$$\delta_{nit} = \begin{cases} 1 & \text{if country } n \text{ is in class } i \text{ for time period } t \\ 0 & \text{else} \end{cases}. \quad (3)$$

For each country,  $n$ , and for each time period  $t$  we observe the country's sovereign debt ratings class  $s_{nt} \in \{1, 2, 3, \dots, C\}$ . Let  $S_{NT} = \{\{s_{nt}\}_{n=1}^N\}_{t=1}^T$  be the information set at time  $T$ . Define  $k_{j0} = \sum_{n=1}^N \delta_{nj0}$  as the number of countries that are in class  $j$  in the initial period and define  $k_{ij} = \sum_{n=1}^N \sum_{t=1}^T \delta_{ni(t-1)} \delta_{njt}$  as the total number of transitions from class  $i$  in time period  $t-1$  to class  $j$  in time period  $t$  across all time periods. The matrix  $\mathcal{K} = [k_{ij}]$  will be referred to as the data transition matrix. Note that if  $T > 2$  it is implicitly assumed that  $\mathbf{P}$  is the same for all  $T-1$  transition periods.

The data density, or likelihood function, for the model defined in (2) is

$$p(S_{NT} | \pi_0, \mathbf{P}) \propto \prod_{i=1}^C \pi_{i0}^{k_{i0}} \prod_{j=1}^C p_{ij}^{k_{ij}} \quad (4)$$

which is the kernel of the product of two independent multivariate Dirichlet (Beta) distributions. Natural conjugate priors for  $\pi_0$  and  $\mathbf{P}$  are also independent Dirichlet distributions defined as

$$p(\pi_0) = \left[ \frac{\Gamma(\sum_{i=1}^C a_{i0})}{\prod_{i=1}^C \Gamma(a_{i0})} \right] \prod_{i=1}^C \pi_{i0}^{(a_{i0}-1)} \quad (5)$$

and

$$p(\mathbf{P}) = \prod_{i=1}^C \left[ \frac{\Gamma(\sum_{j=1}^C a_{ij})}{\prod_{j=1}^C \Gamma(a_{ij})} \right] \prod_{j=1}^C \pi_{ij}^{(a_{ij}-1)}. \quad (6)$$

Here the priors are parameterized by the vector  $a_0 = (a_{10}, \dots, a_{C0})'$  and  $\mathbf{A} = [a_{ij}]$ .

Assuming that the priors are independent then the posterior distribution for (2) is

$$p(\pi_0, \mathbf{P} | S_{NT}) \propto \left[ \frac{\Gamma(\sum_{i=1}^C a_{i0})}{\prod_{i=1}^C \Gamma(a_{i0})} \right] \prod_{i=1}^C \pi_{i0}^{(k_{i0} + a_{i0} - 1)} \prod_{i=1}^C \left\{ \left[ \frac{\Gamma(\sum_{j=1}^C a_{ij})}{\prod_{j=1}^C \Gamma(a_{ij})} \right] \prod_{j=1}^C \pi_{ij}^{(k_{ij} + a_{ij} - 1)} \right\}. \quad (7)$$

The joint posterior density kernel in (7) is the kernel for the product of two Dirichlet distributions. The posterior distribution for  $\pi_0$ , the initial income distribution, is Dirichlet with parameters  $(k_{10} + a_{10}, \dots, k_{C0} + a_{C0})'$ . The posterior distribution for  $\mathbf{P}$  is the product of  $C$  independent Dirichlet distributions with parameters  $(k_{i1} + a_{i1}, \dots, k_{iC} + a_{iC})'$  for  $i = 1, \dots, C$  (Geweke 2005). This posterior distribution is simple to draw directly from so in this instance no Markov chain monte carlo procedure is needed to make draws from the (7). In fact is a simple matter to make identical and independent draws from these independent Dirichlet distributions using the method described in Devroye (1986). Once we have these i.i.d draws from the posterior we can then characterize the exact finite sample distribution of any function of the parameters ( $\pi_0$  and  $A$ ) of the model. Examples of such functions include the measures of overall mobility and measures of directional mobility, which we define in Section 2.3.

## 2.3 Mobility Measures

There are many measures of overall mobility that can be defined. For a complete discussion of the properties and definitions of a large number of mobility measures see Shorrocks (1978) and Geweke et al. (1986). In this paper we report the mobility measure due to Shorrocks (1978),

$$\mathcal{M}_s(\mathbf{P}) = \frac{C - tr(\mathbf{P})}{C - 1}, \quad (8)$$

which is the inverse of the harmonic mean of the expected length of stay in a ratings class, scaled by a factor of  $C/(C - 1)$ . This index satisfies the monotonicity, immobility and strong immobility persistence criteria and hence are internally consistent.<sup>1</sup> This measure

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<sup>1</sup>See Geweke et al. (1986) for a complete discussion on the properties of these mobility indices.

of mobility measures overall mobility and treats movements to higher ratings classes equally with movements to lower ratings classes. We also report conditional mobility measures due to Prais (1955) which report the probability of moving conditional on the initial classification. This conditional measure of mobility is defined as

$$\mathcal{M}_p(j) = \sum_{k=1, k \neq j}^C p_{jk}, \quad (9)$$

for  $j = 1, \dots, C$ .

In the case of bond ratings, movements up the rating distribution have quite different implications to movements down the ratings distribution. Hence we would like to distinguish between the two types of mobility. To do that we use directional mobility measures proposed in Gang et al. (2004). Aggregate measures of upward and downward mobility are

$$\mathcal{M}_U = (C - 1)^{-1} \sum_{j=1}^{C-1} \mathcal{M}_U(j), \quad (10)$$

and

$$\mathcal{M}_D = (C - 1)^{-1} \sum_{j=2}^C \mathcal{M}_D(j). \quad (11)$$

Gang et al. (2004) show that Shorrocks' measure can be decomposed into its upward and downward components. That is,  $\mathcal{M}_S = \mathcal{M}_U + \mathcal{M}_D$  and that these directional mobility measures satisfy directional equivalents of the monotonicity, immobility and strong immobility persistence criterions.

That is, for any transition probability matrix (ratings migration matrix),  $\mathbf{P}_1$ ,  $\mathcal{M}_U(\mathbf{P}_1) \geq 0$ , with the inequality being strict if there are any non-zero elements in the upper-triangular part of  $\mathbf{P}_1$ .<sup>2</sup> Thus the upward mobility measure is positive if there is any probability that a bond will be upgraded to a higher ratings class. Similarly,  $\mathcal{M}_D \geq 0$ ,

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<sup>2</sup>The term "ratings migration matrix" is used extensively in the ratings migration literature and is just the transition probability matrix referred to above. The two terms are used interchangeably in this paper.

with the inequality being strict if there are any non-zero elements in the lower triangular part of  $\mathbf{P}_1$ : the downward mobility measure is positive only if there is a positive probability that a bond will be downgraded to a lower ratings class. Finally, monotonicity implies that for two different ratings migration matrices,  $\mathbf{P}_1$  and  $\mathbf{P}_2$ ,  $\mathcal{M}_U(\mathbf{P}_1) > \mathcal{M}_U(\mathbf{P}_2)$  implies that the ratings migration matrix,  $\mathbf{P}_1$  represents a process that has more upward mobility than the ratings migration process represented by the matrix  $\mathbf{P}_2$ . Similarly,  $\mathcal{M}_D(\mathbf{P}_1) > \mathcal{M}_D(\mathbf{P}_2)$  would imply that the ratings migration process represented by the ratings migration matrix  $\mathbf{P}_1$  would have more downward mobility than the ratings migration process represented by  $\mathbf{P}_2$ .

The second set of directional indices report the probability of moving up or down the distribution conditional on the current class. These indices are:

$$\mathcal{M}_U(j) = \sum_{k=j+1}^M p_{jk} \quad (12)$$

and

$$\mathcal{M}_D(j) = \sum_{k=1}^{j-1} p_{jk}. \quad (13)$$

These two indices describe the probability of moving to a higher (lower) classification in the next period given the state is in classification  $j$  this period. It can also be shown that  $\mathcal{M}_p(j) = \mathcal{M}_U(j) + \mathcal{M}_D(j)$  for  $j = 1, \dots, C$  and that these directional mobility indices satisfy the directional persistence criteria of Geweke et al. (1986).

## 3 Data and Priors

### 3.1 Data

The data are obtained from various issues of *Sovereign Ratings List* published by Moody's Investors Service (henceforth Moody's). We select countries for which a reasonably long time series data for ratings on foreign currency denominated long term bonds are avail-

able. This selection criterion results in a final sample of 92 countries. Of these, 13 are classified as Asian countries, 21 as Latin American countries, 16 as Transition countries of Central and Eastern Europe (including former Soviet Republics), 23 are OECD countries, and 19 as *other*.<sup>3</sup> As discussed elsewhere in this paper, much of our analysis will focus on the comparison of three of these (broadly speaking) geographical groups of countries, namely, Asian countries, Latin American countries, and Transition countries. The industrialized OECD countries act as a benchmark, while *other* is a residual category that is too heterogenous to support any meaningful analysis.

It should be noted that our classification does not adhere to geographical locations and official nomenclature alone, and takes into consideration the relative similarity of the countries with respect to structure and macroeconomic stability of their economies. For example, even though the Middle Eastern countries like Saudi Arabia are Asian countries, as oil producing countries they are structurally different from other Asian countries like China, India and Thailand. Hence, all oil-producing West Asian countries are classified as *other*. Similarly, even though countries like Turkey and the Czech Republic are OECD member countries, the structure and macroeconomic stability their economies are, in general, not comparable with industrialized countries like the United States and Japan. They were certainly not comparable with an average OECD country in 1996, the starting point of our analysis. Hence, while the Central and Eastern European members of the OECD community have been classified under *Transition*, Turkey has been included in the *other* category along with the West Asian countries. The classifications of the countries are reported in Table 1.<sup>4</sup>

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<sup>3</sup>Note that we do not have an African country-category; all the African countries in our sample are part of the *other* category. It was difficult to create a separate African group with just five countries because of the computational problems associated with a large number of empty cells in the transition matrix.

<sup>4</sup>It should be noted that these classifications are not mutually exclusive. For example, Japan is included in both the OECD grouping and the Asian grouping as Japan well fits the description of both classifications. Similarly, Mexico is included both in the OECD sample and the Latin American sample.

Table 1: Countries in our Sample

Asia	Latin America	Transition
CHINA	ARGENTINA	BULGARIA
HONG KONG	BOLIVIA	CROATIA
INDIA	BRAZIL	CZECH REPUBLIC
INDONESIA	CHILE	ESTONIA
JAPAN	COLOMBIA	HUNGARY
KOREA	COSTA RICA	KAZAKHSTAN
MALAYSIA	CUBA	LATVIA
PAKISTAN	DOMINICAN REPUBLIC	LITHUANIA
PHILIPPINES	ECUADOR	MOLDOVA
SINGAPORE	EL SALVADOR	POLAND
TAIWAN	GUATEMALA	ROMANIA
THAILAND	HONDURAS	RUSSIA
VIETNAM	JAMAICA	SLOVAKIA
	MEXICO	SLOVENIA
	NICARAGUA	TURKMENISTAN
	PANAMA	UKRAINE
	PARAGUAY	
	PERU	
	TRINIDAD & TOBAGO	
	URUGUAY	
	VENEZUELA	
OECD	Other	
AUSTRALIA	BAHRAIN	
AUSTRIA	BOTSWANA	
BELGIUM	CYPRUS	
CANADA	EGYPT	
DENMARK	FIJI	
FINLAND	IRAN	
FRANCE	ISRAEL	
GERMANY	JORDAN	
GREECE	LEBANON	
ICELAND	MALTA	
IRELAND	MAURITIUS	
ITALY	MOROCCO	
JAPAN	OMAN	
MEXICO	PAPUA NEW GUINEA	
NETHERLANDS	QATAR	
NEW ZEALAND	SAUDI ARABIA	
NORWAY	SOUTH AFRICA	
PORTUGAL	TUNISIA	
SPAIN	TURKEY	
SWEDEN	U.A.E.	
SWITZERLAND		
UK		
USA		

Table 2: Definition of Ratings Classes

Class	Lowest rating	Highest rating	20-year cumulative default rates
1	C	Caa1	77.198-100.000
2	B3	B1	53.179-61.888
3	Ba3	Ba1	22.919-45.112
4	Baa3	Baa1	6.276-11.355
5	A3	A1	2.267-5.176
6	Aa3	Aa1	0.958-1.560
7	Aaa	Aaa	0.190

### 3.2 Ratings Classifications

Since the model we use is a discrete state Markov chain, we need to define the ratings classifications. Moody's has developed a sophisticated ratings system that can attach one of twenty one possible ratings to a sovereign long term foreign currency bond. The *Aaa* rating indicates that the bond is of exceptionally high credit worthiness and carries minimum credit risk. The credit quality of the bonds decline as we move down the rating scale; *Aa* rated bonds arguably have excellent (but not exceptional) credit worthiness, *A* rated bonds have good credit worthiness, and *Baa* rated bonds have adequate credit worthiness. *Baa3* is the lowest credit rating for an investment grade bond. Below the investment grade threshold, credit worthiness declines in discrete steps from *Ba3* to *C*. Bonds with *Ba* ratings have questionable credit worthiness, those with *B* have poor credit worthiness, and a rating of *Caa* imply very poor credit worthiness. Credit risk is particularly high for *Caa3* through *Caa1* rated bonds, and bonds that are rated *Ca* and *C* are either actually in default or are in default for all practical purposes.<sup>5</sup>

In our analysis, we attempt to strike a balance between the information content of the ratings categories and the practical problem of having classification bins with non-zero observations that is essential for the analysis. Our classifications are reported in Table 2.

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<sup>5</sup>For details about ratings classification and a discussion about the factors that affect sovereign credit ratings, see Cantor & Packer (1996).

We make the reasonable assumption that the numeric part of all alphanumeric ratings, e.g.,  $3$  for a  $Baa3$  rating, are refinements of the (basic) alphabetical ratings (i.e.,  $Baa$ ). This simplifying assumption is consistent with the classifications used by Altman (1998), and is also supported by the the cumulative 20-year default rates of bonds during the 1983-2006 period, which we report in table.<sup>6</sup> It is easily seen that the ranges of default rates are non-overlapping across our ratings classes, such that our ratings classes are mutually exclusive. Our only innovation is to merge the alphabetical ratings categories  $Caa$  and  $C$  into class 1, essentially arguing that there is not much difference between bonds that have very high default probability and those that are actually in default. This is consistent with the comparable cumulative default rates of 73.48% and 78.46% for  $Caa2$  (the median of the  $Caa$  alphabetical category) and  $C$  rated bonds, respectively.

### 3.3 Priors

We define natural conjugate priors for the parameters  $\pi_0$  and  $P$ . In fact we define  $C + 1$  independent prior distributions for  $\pi_0$  and the  $C$  rows of  $P$ . Each prior has the same form. The general form of the priors for  $\pi_0$  and for the  $j^{th}$  row of  $P$ ,  $P(j)$ , is defined in (14),

$$\begin{aligned} (\pi_{10}, \dots, \pi_{C0}) &\sim Di_M(a_0) \\ (p_{j1}, \dots, p_{jM}) &\sim Di_M(A(j)) \quad j = 1, \dots, M. \end{aligned} \tag{14}$$

where  $Di_M(a_0)$  and  $Di_M(A(j))$  refer to a multivariate-Beta (Dirichlet) of order  $M - 1$  indexed by the parameter vector  $a_0 = (a_1, \dots, a_M)$ , and  $A(j) = (A_{j1}, \dots, A_{jM})$ , respec-

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<sup>6</sup>See Exhibit 26 of *Corporate default and recovery rates, 1920-2006*, Moody's Investors Service, February 2007.

tively. The multivariate-Beta distribution of order  $M - 1$  has density

$$p(x|a) = \frac{\Gamma\left(\sum_{j=1}^M a_j\right)}{\prod_{j=1}^M \Gamma(a_j)} \prod_{j=1}^M x_j^{(a_j-1)} \quad (15)$$

where  $a_j > 0$  for all  $j = 1, \dots, M$  and  $x \in \{x : x_j > 0 \ (j = 1, \dots, M), \sum_{j=1}^M x_j = 1\}$ . The priors defined are therefore indexed by the vector  $a_0$  and the matrix  $A$ . This prior has a notional sample interpretation in that we can interpret the values of  $a_0$  to be the observations from a notional data set with  $a_{01}$  observations in the first ratings class,  $a_{02}$  observations in the second ratings class and so on. This notional interpretation is nice in the sense that the smaller are the values in  $a_0$  the less influence they have on the posterior distribution.

The prior in this context has two important uses. First, it allows us to explicitly state our prior beliefs about the parameters of the model. Second, it allows us to “fill in” zero elements of the data transition matrix,  $\mathcal{K}$ . In our application, with classification definitions given in Table 2, we do not observe any transition from classification 1 to classification 7 in any single period. Hence, the data transition matrix,  $\mathcal{K}$ , has a zero in the 7<sup>th</sup> column of the 1<sup>st</sup> row. In the definition of the Dirichlet distribution of order  $M - 1$  given in (15) we see that the parameter vector that indexes the distribution cannot contain any zero elements. The posterior distribution, given in (7), is the product of  $M + 1$  Dirichlet distribution where the posterior of the  $j^{th}$  row of  $P$ ,  $P(j)$ , is indexed by  $A(j) + \mathcal{K}(j)$ , the sum of the  $j^{th}$  rows of  $A$  and  $\mathcal{N}$ . Thus, as long as the prior parameter,  $A(j)$ , does not contain any 0's, the posterior parameter will not contain any zero elements.

The priors used in this paper are designed to reflect our uncertainty over the parameters of the model. The first parameter is the initial ratings distribution. The prior chosen for this parameter is indexed by  $a_0 = (0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1)$ . Thus the prior initial distribution is a uniform distribution in that each rating class is equally likely in the prior. The fact that each value of  $a_0$  is small reflects our desire to have the data drive

the posterior distribution. In a notional prior context the prior as stated implies that the prior comes from a notional data set with only 0.1 “observations” in each ratings class. Thus every observation from the data set is weighted ten times as much as our “observations” from the notional prior data set. The prior for  $P$  is indexed by  $A$ . Each row of  $A$  refers to an independent prior for the corresponding row of  $P$ . Each row of  $A$  was chosen so that the probability of staying in the current ratings class is 3.5 times more likely than the probability of moving to another class. For example the first row of  $A$  is (0.21, 0.0003, 0.0003, 0.0003, 0.0003, 0.0003, 0.0003). This prior places most of the prior probability on the main diagonal of  $P$ . This prior yields a prior probability of not moving of 0.99 and a combined prior probability of moving of 0.01 for each ratings class. Again that numbers were chosen to be small relative to the number of observed transitions so that the calculated posterior is driven mainly by the observed data. The other nice consequence of this prior is that the prior represents a notional data set where there is almost no change in the ratings classes of the “observations”. This implies that the posterior distribution of the mobility scores will be driven by observed movements in actual debt ratings rather than movements in debt ratings from the notional prior distribution.

## 4 Empirical Results: Overall vs. Upward and Downward Mobility Scores

In this section we report mobility indices for our whole sample of 92 countries and for various interesting sub-groups of countries. We report mobility indices for transitions from 1996 to 2005, as well as for the sub-periods 1996-1999, 2000-2002, and 2002-2005. For each time period we report the Shorrocks’ overall measure of mobility and the decomposition of the Shorrocks measure into its directional components. We also report the conditional (Prais) measures of mobility for each ratings class together with it’s

directional decomposition.

Table 3 reports mobility indices for the full sample of countries. The table is broken up into four sections, one for each time period that we look at. Looking first at ratings mobility of the full sample of countries, we see that the mobility is similar for the full sample period (1996-2005) and for the first two sub-periods of 1996-1999 and 1999-2002. The overall mobility for the whole sample is 0.126 while for the first two sub-periods it is 0.132 and 0.138 respectively. Only for the last period (2002-2005) is there a significant difference with the overall mobility significantly lower at 0.09. However when we look at the directional mobility indices we see that mobility during the sub-periods are quite different. For the first sub-period (1996-1999) the downward mobility score is higher than the upward mobility score suggesting that countries were more likely to be downgraded during this period than upgraded. This result turns around sharply during the second period (1999-2002) where the upward mobility score is ten times the downward mobility score. This suggests that, whereas the period from 1996-1999 was a period of financial stress, the period from 1999-2002 was a period of recovery where countries who were downgraded earlier were now upgraded.

The conditional (Prais) mobility scores provide deeper insight into these ratings migration pattern. In the period from 1996-1999, most of the (downward) action takes place in ratings classes 3, 4 and 5, i.e., those just below or just above investment grade. Here we see the conditional downward mobility scores being significantly higher than the upward scores. Thus it appears that the countries that were likely to be downgraded were emerging markets as opposed to mature industrialized economies. In addition, we see that the probability of moving out of the lowest ratings class is much lower in the first period than the subsequent periods while the probability of downgrade of *Aa* and *Aaa* rated bonds remained roughly similar across time periods. This goes on to suggest that it is the marginally investment grade and below investment grade countries that are driving the result.

Table 3: Mobility Measures: Full Sample

Mobility Measure	1996-2005			1996-99		
	Overall	Upward	Downward	Overall	Upward	Downward
Shorrocks	0.126 (0.015)	0.088 (0.013)	0.038 (0.007)	0.132 (0.023)	0.054 (0.016)	0.079 (0.017)
Prais						
Class 1	0.121 (0.056)	0.121 (0.056)	— —	0.003 (0.039)	0.003 (0.039)	— —
Class 2	0.134 (0.030)	0.074 (0.023)	0.060 (0.021)	0.131 (0.060)	0.065 (0.044)	0.066 (0.044)
Class 3	0.175 (0.032)	0.088 (0.023)	0.088 (0.024)	0.209 (0.055)	0.037 (0.026)	0.172 (0.050)
Class 4	0.126 (0.026)	0.084 (0.021)	0.042 (0.016)	0.104 (0.040)	0.034 (0.024)	0.069 (0.033)
Class 5	0.057 (0.023)	0.029 (0.016)	0.028 (0.016)	0.213 (0.081)	0.087 (0.060)	0.126 (0.066)
Class 6	0.134 (0.036)	0.134 (0.036)	0.000 (0.001)	0.097 (0.044)	0.097 (0.044)	0.000 (0.000)
Class 7	0.008 (0.009)	— —	0.008 (0.009)	0.037 (0.037)	— —	0.037 (0.037)
Mobility Measure	1999-2002			2002-05		
	Overall	Upward	Downward	Overall	Upward	Downward
Shorrocks	0.138 (0.024)	0.126 (0.024)	0.012 (0.006)	0.090 (0.019)	0.060 (0.016)	0.030 (0.010)
Prais						
Class 1	0.139 (0.090)	0.139 (0.090)	— —	0.110 (0.068)	0.110 (0.068)	— —
Class 2	0.109 (0.042)	0.056 (0.032)	0.054 (0.028)	0.167 (0.052)	0.106 (0.043)	0.060 (0.033)
Class 3	0.146 (0.052)	0.146 (0.052)	0.000 (0.001)	0.159 (0.059)	0.080 (0.044)	0.080 (0.043)
Class 4	0.162 (0.047)	0.145 (0.045)	0.017 (0.017)	0.103 (0.045)	0.062 (0.036)	0.041 (0.027)
Class 5	0.035 (0.032)	0.034 (0.032)	0.000 (0.002)	0.000 (0.001)	0.000 (0.001)	0.000 (0.001)
Class 6	0.236 (0.069)	0.235 (0.069)	0.000 (0.002)	0.000 (0.005)	0.000 (0.000)	0.000 (0.005)
Class 7	0.000 (0.003)	— —	0.000 (0.003)	0.000 (0.001)	— —	0.000 (0.001)

We now divide the countries into their sub-categories in an attempt to understand the results obtained for the group of countries as a whole. First we look at the mobility scores for the sub-group of Asian countries that are reported in Table 4. Looking first at the full sample period (1996-2005), we see that the overall mobility score for the Asian countries (0.180) is higher than the overall mobility score for the full sample of countries (0.126), suggesting that the former were experienced more ratings migration during this period than the sample countries as a whole. We also see that the overall mobility for Asian countries is equally divided into upward and downward mobility. By contrast, for the full sample of countries upward mobility and downward mobility contributed roughly two-thirds and one-third of the overall score respectively. The highest mobility occurs in ratings classes 3 and 4 and upward and downward mobility scores contribute equally to the overall mobility of these ratings classes.

However when we break the sample into three distinct time-periods a richer story emerges. In the first sub-period (1996-1999), we see that there is more overall mobility for the Asian countries (0.246) than the 1996-2005 average (0.180) but that this mobility is almost entirely downward mobility (0.229). The individual classes that suffer the biggest mobility are classes 4, 5, and 7 with downward mobility scores significantly higher than the average. In the second sub-period (1999-2002) the direction of mobility is reversed. The overall mobility is lower (0.119) compared to the first sub-period (0.246). Further, the composition of the mobility during this period is also quite different with almost all of the mobility being upward mobility. The individual classes that have the highest mobility in the second period are classes 3, 4, and 6 suggesting that many of the down-gradings that occurred in the previous period were reversed in this period.

The last sub-period (2002-2005) has a very similar overall mobility to the second sub-period (0.104), and the break-down of this mobility is such that two-thirds of the overall mobility is upward mobility and one-third of the overall mobility is downward mobility. In other words, the ratings recovery of the Asian countries continued into the

Table 4: Mobility Measures: Asian Countries

Mobility Measure	1996-2005			1996-99		
	Overall	Upward	Downward	Overall	Upward	Downward
Shorrocks	0.180 (0.043)	0.096 (0.034)	0.084 (0.027)	0.246 (0.064)	0.017 (0.016)	0.229 (0.064)
Prais						
Class 1	0.193 (0.161)	0.193 (0.161)	— —	0.004 (0.040)	0.004 (0.040)	— —
Class 2	0.097 (0.063)	0.051 (0.047)	0.047 (0.046)	0.194 (0.158)	0.000 (0.004)	0.193 (0.158)
Class 3	0.273 (0.095)	0.136 (0.070)	0.137 (0.073)	0.274 (0.129)	0.096 (0.085)	0.178 (0.110)
Class 4	0.192 (0.084)	0.096 (0.064)	0.096 (0.062)	0.399 (0.202)	0.000 (0.006)	0.399 (0.202)
Class 5	0.115 (0.060)	0.000 (0.000)	0.115 (0.060)	0.281 (0.130)	0.000 (0.005)	0.281 (0.130)
Class 6	0.100 (0.066)	0.100 (0.066)	0.000 (0.001)	0.000 (0.004)	0.000 (0.002)	0.000 (0.004)
Class 7	0.107 (0.096)	— —	0.107 (0.096)	0.324 (0.228)	— —	0.324 (0.228)
Mobility Measure	1999-2002			2002-05		
	Overall	Upward	Downward	Overall	Upward	Downward
Shorrocks	0.119 (0.048)	0.117 (0.047)	0.001 (0.010)	0.104 (0.042)	0.070 (0.037)	0.034 (0.028)
Prais						
Class 1	0.260 (0.200)	0.260 (0.200)	— —	0.008 (0.061)	0.008 (0.061)	— —
Class 2	0.001 (0.008)	0.001 (0.008)	0.000 (0.000)	0.097 (0.088)	0.097 (0.088)	0.000 (0.001)
Class 3	0.129 (0.117)	0.128 (0.117)	0.001 (0.011)	0.405 (0.205)	0.205 (0.167)	0.200 (0.165)
Class 4	0.110 (0.097)	0.110 (0.097)	0.000 (0.002)	0.111 (0.097)	0.110 (0.097)	0.000 (0.008)
Class 5	0.001 (0.014)	0.001 (0.013)	0.000 (0.006)	0.001 (0.016)	0.000 (0.006)	0.001 (0.015)
Class 6	0.206 (0.123)	0.206 (0.123)	0.000 (0.005)	0.001 (0.013)	0.000 (0.007)	0.001 (0.012)
Class 7	0.007 (0.056)	— —	0.007 (0.056)	0.001 (0.007)	— —	0.001 (0.007)

new century, albeit at a slower pace.

Another grouping of countries that faced economic crises during the sample period are the Latin American countries. The sovereign rating mobility scores for these countries are reported in Table 4. For the full sample period (1996-2005), as well as for the first two sub-periods, we see that the Latin American countries have fairly low overall mobility, approximately half of which is the contribution of each of upward and downward mobility. The differences in the directional mobility scores is not statistically significant for any of these three time periods. This is in sharp contrast with the much higher overall mobility scores for the Asian countries in the first two sub-periods. As such, this implies that there was no ratings contagion from the Asian countries to the Latin American countries during 1996-1999 and, consequently, no ratings rebound among the latter during 1999-2002 either.

A plausible explanation for the absence of ratings contagion across these two regions dominated by emerging markets is that the ratings of most Latin American countries were low to begin with, typically below the investment grade, and hence, in the absence of catastrophic events, there was not much scope significant further downgrade in 1996-1999. Table 6 reports the posterior distribution of the initial distribution of sovereign debt ratings for Latin American countries for each starting period of our sub-samples. It is easily seen that during both 1996 and 1999 more than 70% of these countries had ratings below the investment grade.

In the immediate aftermath of the crisis in Argentina (2002-2005), the ratings mobility of Latin American countries more than doubled compared with the corresponding number of 1999-2002, from 0.045 to 0.091. However, this overall mobility score for 2002-2005 is not significantly different from the 1996-1999 score of 0.077. Hence, in the absence of the ability to decompose the overall score into upward and downward mobility scores, it would seem that ratings migration in Latin America for 1996-1999 and 1999-2002 are

Table 5: Mobility Measures: Latin American Countries

Mobility Measure	1996-2005			1996-99		
	Overall	Upward	Downward	Overall	Upward	Downward
Shorrocks	0.077 (0.021)	0.037 (0.014)	0.041 (0.016)	0.077 (0.031)	0.032 (0.020)	0.044 (0.024)
Prais						
Class 1	0.087 (0.057)	0.087 (0.057)	— —	0.007 (0.066)	0.007 (0.066)	— —
Class 2	0.158 (0.047)	0.070 (0.032)	0.088 (0.037)	0.189 (0.093)	0.128 (0.078)	0.061 (0.058)
Class 3	0.138 (0.050)	0.059 (0.033)	0.079 (0.038)	0.168 (0.087)	0.056 (0.054)	0.112 (0.074)
Class 4	0.067 (0.037)	0.000 (0.001)	0.067 (0.037)	0.079 (0.071)	0.000 (0.002)	0.079 (0.071)
Class 5	0.005 (0.049)	0.002 (0.033)	0.003 (0.037)	0.005 (0.050)	0.002 (0.026)	0.003 (0.042)
Class 6	0.005 (0.048)	0.001 (0.030)	0.003 (0.038)	0.006 (0.054)	0.001 (0.027)	0.005 (0.047)
Class 7	0.005 (0.049)	— —	0.005 (0.049)	0.006 (0.059)	— —	0.006 (0.059)
Mobility Measure	1999-2002			2002-05		
	Overall	Upward	Downward	Overall	Upward	Downward
Shorrocks	0.045 (0.023)	0.025 (0.015)	0.020 (0.017)	0.091 (0.032)	0.031 (0.017)	0.060 (0.026)
Prais						
Class 1	0.000 (0.005)	0.000 (0.005)	— —	0.130 (0.083)	0.130 (0.083)	— —
Class 2	0.087 (0.057)	0.043 (0.040)	0.044 (0.043)	0.199 (0.088)	0.052 (0.049)	0.148 (0.077)
Class 3	0.103 (0.070)	0.103 (0.070)	0.000 (0.001)	0.134 (0.084)	0.000 (0.006)	0.134 (0.084)
Class 4	0.061 (0.056)	0.000 (0.001)	0.061 (0.056)	0.063 (0.058)	0.000 (0.003)	0.063 (0.058)
Class 5	0.006 (0.057)	0.002 (0.032)	0.004 (0.047)	0.006 (0.055)	0.003 (0.037)	0.004 (0.041)
Class 6	0.005 (0.047)	0.001 (0.011)	0.005 (0.046)	0.006 (0.048)	0.001 (0.015)	0.005 (0.045)
Class 7	0.005 (0.047)	— —	0.005 (0.047)	0.006 (0.056)	— —	0.006 (0.056)

Table 6: Posterior Moments of Initial Ratings Distribution: Latin American Countries

Year	Class 1	Class 2	Class 3	Class 4	Class 5	Class 6	Class 7
1996	0.0093 (0.0295)	0.3166 (0.1392)	0.4202 (0.1507)	0.2221 (0.1323)	0.0099 (0.0297)	0.0106 (0.0308)	0.0112 (0.0346)
1999	0.0947 (0.0589)	0.3757 (0.0989)	0.3260 (0.0943)	0.1882 (0.0794)	0.0055 (0.0176)	0.0051 (0.0147)	0.0048 (0.0150)
2002	0.1420 (0.0736)	0.3261 (0.0996)	0.2869 (0.0940)	0.2290 (0.0843)	0.0056 (0.0185)	0.0061 (0.0202)	0.0043 (0.0135)

largely comparable. However, we can see that while the upward mobility scores for these two periods are almost identical, the downward mobility score for 2002-2005 (0.060) is 36% higher than the corresponding score for 1996-2002 (0.044). Once again, our results demonstrate the need to be able to decompose overall ratings migration scores into their upward and downward components.

Next, we concentrate on an interesting group of countries that also went through a large amount of economic upheaval during the sample period, namely, the Transition economies of the former socialist economies of Central and Eastern Europe, and the former Soviet Republics. The mobility scores for these countries are reported in Table 7.

The overall mobility score for the Transition countries for the entire 1996-2005 period (0.146) is comparable to that of the Asian countries for the same time period (0.180). However, while the upward mobility score of the former countries (0.146) is about 40% higher than the downward score. By contrast, the upward and downward mobility scores of the Asian countries are roughly similar (0.096 and 0.084 respectively). The contrast between the countries is even more stark when we have to take a closer look at ratings migration within the three sub-periods. In the first sub-period (1996-1999), while the downward mobility scores for both Asian and Transition countries are much higher than the respective upward mobility scores, the downward mobility score of the Asian countries (0.229) is nearly four times that of the Transition countries (0.062). Similarly, in the second sub-period (1999-2002), the upward mobility score of the Transition countries

Table 7: Mobility Measures: Transition Countries

Mobility Measure	1996-2005			1996-99		
	Overall	Upward	Downward	Overall	Upward	Downward
Shorrocks	0.146 (0.031)	0.104 (0.027)	0.041 (0.016)	0.063 (0.028)	0.001 (0.009)	0.062 (0.026)
Prais						
Class 1	0.137 (0.116)	0.137 (0.116)	— —	0.006 (0.055)	0.006 (0.055)	— —
Class 2	0.186 (0.067)	0.124 (0.056)	0.062 (0.040)	0.001 (0.009)	0.001 (0.008)	0.000 (0.004)
Class 3	0.317 (0.091)	0.159 (0.074)	0.157 (0.072)	0.303 (0.126)	0.001 (0.006)	0.303 (0.126)
Class 4	0.197 (0.062)	0.173 (0.059)	0.024 (0.023)	0.057 (0.052)	0.000 (0.002)	0.056 (0.052)
Class 5	0.032 (0.030)	0.032 (0.030)	0.000 (0.002)	0.001 (0.012)	0.001 (0.007)	0.001 (0.009)
Class 6	0.001 (0.011)	0.000 (0.006)	0.001 (0.010)	0.002 (0.040)	0.000 (0.001)	0.002 (0.040)
Class 7	0.005 (0.042)	— —	0.005 (0.042)	0.008 (0.066)	— —	0.008 (0.066)
Mobility Measure	1999-2002			2002-05		
	Overall	Upward	Downward	Overall	Upward	Downward
Shorrocks	0.229 (0.054)	0.207 (0.051)	0.023 (0.018)	0.081 (0.037)	0.080 (0.034)	0.001 (0.010)
Prais						
Class 1	0.244 (0.190)	0.244 (0.190)	— —	0.001 (0.009)	0.001 (0.009)	— —
Class 2	0.247 (0.103)	0.123 (0.077)	0.125 (0.080)	0.224 (0.134)	0.224 (0.134)	0.000 (0.000)
Class 3	0.374 (0.161)	0.374 (0.161)	0.000 (0.002)	0.156 (0.130)	0.156 (0.130)	0.000 (0.007)
Class 4	0.374 (0.115)	0.374 (0.115)	0.000 (0.003)	0.098 (0.093)	0.098 (0.093)	0.000 (0.001)
Class 5	0.124 (0.106)	0.124 (0.106)	0.000 (0.005)	0.000 (0.003)	0.000 (0.001)	0.000 (0.003)
Class 6	0.005 (0.046)	0.001 (0.023)	0.004 (0.040)	0.001 (0.006)	0.000 (0.002)	0.001 (0.006)
Class 7	0.007 (0.064)	— —	0.007 (0.064)	0.006 (0.057)	— —	0.006 (0.057)

(0.207) is not just (nearly) ten times the downward mobility score,<sup>7</sup> it is also nearly double the upward mobility score of the Asian countries (0.117). Finally, in the third sub-period (2002-2005), while almost the entire mobility score for the Transition countries (0.081) can be accounted for by upward mobility (0.080),<sup>8</sup> the downward mobility score for the Asian countries (0.034) is about 50% of the upward mobility score.

The need to look at upward and downward mobility separately, however, becomes most apparent when we compare the mobility scores for the Latin American and the Transition countries. In the first sub-period, the overall mobility scores for Latin American countries (0.077) and the Transition countries (0.063) are similar and the difference is not statistically significant. However, while the upward and downward mobility scores for the former countries for this period are roughly the same (0.032 and 0.044 respectively), downward mobility in Transition countries is mostly accounted for by downward mobility (0.062). Similarly, in the third sub-period, the overall mobility scores for these two groups of countries are not much different, 0.091 for Latin American countries and 0.081 for Transition countries. However, while downward mobility accounts for about two-thirds of this mobility in Latin America, upward mobility accounts for nearly all of the mobility among the Transition countries.

Finally we investigate the ratings mobility of the OECD countries. These mobility scores are reported in Table 8. We can see that the overall mobility score of 0.127 of these industrialized countries for the full sample period (1996-2005) is similar to those of the Transition economies (0.146) and somewhat lower than that of the Asian countries (0.180). As with the Transition countries, upward mobility accounts for most of this overall mobility score for the OECD countries.<sup>9</sup> However, while the downward mobility

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<sup>7</sup>With the sole exception of the countries in ratings class 2, sovereign debt ratings almost always improved for Transition economies during this period. The most amount of action for this period occurred for those countries that were initially in either ratings class 3 or 4. They had identical probabilities of moving to a higher ratings class of 0.374.

<sup>8</sup>However, unlike in the second sub-period, where the majority of the movement was from bonds that were rated in ratings classes 3, 4, and 5, the majority of the movement in third sub-period (2002-2005) was in ratings class 2.

<sup>9</sup>Overall, it appears that the OECD sovereign debt ratings were not affected by the economic crises

still accounts for 28% of the overall mobility score of the Transition economies, it accounts for less than 1% of the overall mobility in the Transition countries. We can make a similar observation about the contrasts between the OECD and Asian countries during the first sub-period. In 1996-1999, the overall mobility score of the Asian countries (0.246) is nearly the same as that of the OECD countries (0.229). However, while downward mobility accounts for 93% of the overall mobility score for the former, it accounts for less than 1% of the mobility score for the latter. Our results once again highlight the analytical shortcomings of a single overall ratings mobility score, and the importance of having separate estimates for upward and downward mobility.

In this section, we have successfully demonstrated the need for separate upward and downward mobility scores and, correspondingly, the shortcoming of an overall mobility score in painting an accurate picture about ratings migration patterns, especially when the sample of countries (or bond issuers) is heterogeneous in nature. One final question that remains is whether the upward and downward mobility scores estimated using our algorithm are sensible as well. As discussed earlier in this paper, our estimates suggest the following, among others: (a) A large number of Asian countries experienced ratings downgrade in the wake of the 1997 crisis, but their ratings bounced back shortly thereafter. (b) The ratings of the Latin American countries, which were largely non-investment grade to begin with, were not significantly affected by the Asian crisis. However, these countries did not benefit from the subsequent upward mobility in Asian (and also Transition) countries. (c) The Transition economies, which were anticipating accession to the European Union, and the macroeconomic stability associated with the membership, experienced continued ratings upgrade during much of the period. (d) Countries in the

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in Asia and Latin America (except of course for those countries that are also included in the Asian and Latin American groups). Further, it appears that the sovereign debt ratings of the weaker OECD countries have generally improved over time to the extent that there is now very little ratings mobility in the sovereign debt ratings for these rich developed countries. Given that most of them were in ratings classes 5, 6 and 7 by 2002, the conditional likelihood of upward mobility was low, and these countries were evidently able to deal with factors like rising commodity prices without much of an adverse impact on their credit worthiness.

Table 8: Mobility Measures: OECD Countries

Mobility Measure	1996-2005			1996-99		
	Overall	Upward	Downward	Overall	Upward	Downward
Shorrocks	0.127 (0.044)	0.126 (0.044)	0.001 (0.002)	0.229 (0.049)	0.222 (0.048)	0.006 (0.007)
Prais						
Class 1	0.006 (0.073)	0.006 (0.073)	— —	0.005 (0.066)	0.005 (0.066)	— —
Class 2	0.004 (0.054)	0.004 (0.053)	0.000 (0.007)	0.005 (0.060)	0.004 (0.058)	0.000 (0.012)
Class 3	0.233 (0.181)	0.232 (0.180)	0.000 (0.004)	0.001 (0.014)	0.001 (0.012)	0.000 (0.006)
Class 4	0.119 (0.103)	0.119 (0.103)	0.000 (0.001)	0.302 (0.221)	0.302 (0.221)	0.001 (0.010)
Class 5	0.238 (0.136)	0.238 (0.136)	0.000 (0.001)	0.905 (0.168)	0.905 (0.168)	0.000 (0.002)
Class 6	0.155 (0.042)	0.154 (0.042)	0.000 (0.000)	0.117 (0.054)	0.116 (0.054)	0.000 (0.001)
Class 7	0.008 (0.008)	— —	0.008 (0.008)	0.037 (0.037)	— —	0.037 (0.037)
Mobility Measure	1999-2002			2002-05		
	Overall	Upward	Downward	Overall	Upward	Downward
Shorrocks	0.184 (0.046)	0.184 (0.046)	0.000 (0.006)	0.002 (0.016)	0.002 (0.015)	0.000 (0.005)
Prais						
Class 1	0.006 (0.074)	0.006 (0.074)	— —	0.003 (0.052)	0.003 (0.052)	— —
Class 2	0.005 (0.064)	0.004 (0.055)	0.001 (0.032)	0.006 (0.071)	0.005 (0.064)	0.001 (0.032)
Class 3	0.838 (0.246)	0.838 (0.247)	0.000 (0.008)	0.003 (0.044)	0.003 (0.043)	0.000 (0.006)
Class 4	0.001 (0.016)	0.001 (0.016)	0.000 (0.000)	0.000 (0.008)	0.000 (0.008)	0.000 (0.001)
Class 5	0.001 (0.014)	0.000 (0.002)	0.000 (0.014)	0.000 (0.001)	0.000 (0.000)	0.000 (0.001)
Class 6	0.255 (0.083)	0.255 (0.083)	0.000 (0.000)	0.000 (0.001)	0.000 (0.000)	0.000 (0.001)
Class 7	0.000 (0.000)	— —	0.000 (0.000)	0.000 (0.000)	— —	0.000 (0.000)

ratings classes 2, 3 and 4 were disproportionately more likely to experience downward ratings mobility than their counterparts in the investment grade categories. These results are consistent with the experiences of the actual countries in our samples.

## 5 Conclusion

In this paper, we have estimated a discrete-state first order Markov chain model of (sovereign) debt ratings. We use data on a large cross-section of countries over a ten year period to estimate ratings migration matrices for the whole sample of countries and for different sub-periods and sub-samples of the countries. As in the existing literature we find that the assumption that the debt ratings migration matrix is constant across time is too strong of an assumption; the Markov chain is not time-homogenous. We also find that the model is not homogenous in the cross-sectional dimension; there are significant differences between the ratings migration matrices for different sub-groups of the countries in our sample.

While some of the conclusions of the non-homogeneity of the basic Markov model can be made from standard mobility scores alone we show that it is important to be able to decompose the observed mobility into it's directional components. We use an existing decomposition of a well-known overall mobility score and show that this decomposition allows us to better understand the underlying dynamics of the debt ratings migration for the countries in our sample. In particular, we show that in some cases while the overall mobility scores between periods are almost identical the directional mobility scores are starkly different, both in a statistical sense and in an economic sense. We also show that even when we observe different overall mobility between either two time periods of two sub-groups of our sample the directional mobility scores help us better explain what exactly is different between the two samples or periods. This allows us to better characterize the qualitative properties of the different sub-groups of sovereign debt and

allow us to quantify and sign the change in the quality of the sovereign debt over time and between sub-groups of the sample.

Our methodology also allows us to similarly decompose conditional overall mobility scores, i.e., the mobility scores for different ratings classes. Indeed, even if the sample size is small such that some ratings classes have zero observations at a given point in time, we can overcome the computational problems by using priors for those ratings classes that are arbitrarily close to (but not equal to) zero. At the same time, our methodology ensures that the posterior moments for the ratings classes are generated almost entirely on the basis of the data, with very little weights attached to the non-zero priors. However, while we report the conditional mobility scores for the full sample as well as the sub-samples, for all time periods, in the interest of brevity, we do not full discuss those results, which reinforce our main findings.

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